The Location of MH370

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September 25, 2014

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Executive Summary

This white paper demonstrates the precise location of MH370's route after radar contact was lost, including its impact point at the 7th handshake arc. Flight routes that were previously considered possibilities are excluded by applying a restriction on the allowable variation in true air speed. Additional restrictions, based on average true air speed and on the time of the final turn, allow the identification of the final selected waypoint and result in a determination of the 7th arc crossing with an uncertainty of less than 15 nautical miles.

I developed a computer-based model to generate, evaluate, and optimize flight routes. This model includes high-altitude wind speed, wind direction, and magnetic declination as functions of aircraft location. By evaluating all possible lateral navigation modes over the full range of plausible flight directions, I identified the unique route that is consistent with the satellite BTO and BFO data and with the additional constraints on true air speed and turn time based on the radar track and the satellite data.

The identified MH370 route is a great circle passing directly over Maimun Saleh Airport on Weh Island.

A route passing directly over Maimun Saleh Airport is the route flown by the MH370 flight crew (after radar contact is lost) for the following reasons:

1. ***it goes to a plausible, nearby destination (the nearest airport),***
2. ***it is a great circle route flown using the Flight Management System (the normal method),***
3. ***it is an extremely simple route, requiring only one turn and no speed changes,***
4. ***it matches the handshake arcs derived from the BTO data very well (< 1.5 NM RMS error),***
5. ***it has extremely steady true air speed as expected (~1 knot variability),***
6. ***it matches the average true air speed derived from the radar track after diversion (within 1 %),***
7. ***it matches the BFO data for the entire flight (~ 5 Hz residual RMS error), and***
8. ***it matches the southward turn completion time derived from the BFO data at 18:28:46 UTC (within 20 seconds).***

The MH370 impact point is the 7th handshake arc crossing at (40.24S, 83.53E).

No route flyable on autopilot with very steady air speed ends within the current ATSB Priority Search Area.

The MH370 impact zone is located ~900 nautical miles southwest of the center of the current ATSB Priority Search Area.

Based on the results of this analysis, I recommend a new search zone (32 NM long by 20 NM wide) that covers the 00:19 UTC handshake arc from 83.2-83.8 degrees east longitude and has an area of 2,200 km^2.

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1 Introduction

The disappearance of Flight MH370 is a mystery unparalleled in aviation history.

Solving this mystery requires knowledge of the five basic story elements – the who, what, when, where, and why. The answers to the first three questions (who, what, and when) are already known. In this case, we must first know the "where" before we can understand the "why." This white paper answers the "where" question.

1.1 **Purpose**

The purpose of this white paper is to present evidence that MH370 flew a very particular route after radar contact was lost on the night of March 7-8, 2014. The information needed to identify that route is contained in the reports of the Inmarsat satellite data and the radar track summarized by the Australian Transport Safety Bureau. Knowing the route allows precise determination of the impact point at the 7th handshake arc crossing. Knowing the impact point allows the wreckage field to be located. Recovery of the Flight Data Recorder and Cockpit Voice Recorder may then provide additional clues to the cause of this disaster. Subsequent recovery of aircraft wreckage may allow a determination of the exact nature of the malfunction which appears to have disabled the flight crew.

Two communities are served by this effort. First, grieving family members want to know what happened to their loved ones and why it happened. Second, the safety of the flying public is compromised until we know the cause of this disaster and take steps to prevent its recurrence.

1.2 **Basis of Analysis**

The basis for this analysis is two reports relating to MH370:

1) Malaysia Department of Civil Aviation "MH370 Data Communication Logs" issued on May 27, 2014 (referred to as "MH370 Data Communication Logs"), and
2) Australian Transport Safety Bureau "MH370 – Definition of Underwater search Areas" (Report AE-2014-054) issued June 26, 2014 and revised August 18, 2014 (referred to here as "ATSB Report").

In this white paper I will refer to the aircraft by its registration number "9M-MRO," and I will refer to the flight or flight route by its number "MH370."

The ATSB Report describes three analyses (called A, B, and C) of possible flight routes with varying assumptions. My analysis uses assumptions which are most similar to those used by the ATSB in Analysis A plus several additional criteria. My results can be used to eliminate from consideration essentially all the routes predicted by Analysis B and by Analysis C and almost all the routes predicted by Analysis A. The exclusion of those routes shifts the most-likely impact zone to a point outside the southwest end of the ATSB Wide Area.

1.3 **New Analysis Method**

Previous analyses have generated routes based on simultaneous fits to the Burst Timing Offset (BTO) and the Burst Frequency Offset (BFO) data. My approach is different. I start with the BTO data and generate routes that have very "steady" (i.e., constant) true air speed. An accurate wind model is necessary for this method to succeed in achieving a precise result with very little error in the predicted impact zone. Fortunately, such wind data are available and have been incorporated into my computer model.

Here are the principal steps in my analysis process:

1. I first find the routes (consistent with the BTO data) having "steady true air speed" by evaluating all Lateral Navigation modes in all plausible directions.
2. After I select the best fits to the BTO data with steady true air speed, I check for consistency with the BFO data from 18:40 to 00:19 UTC.
3. The next test is to check the average true air speed for consistency with the average true air speed measured during the radar track between the times of the emergency diversion turn-around and the final military radar contact.
4. The final test is to check for consistency with the BFO data between 18:25 and 18:40 UTC, during which time the final southward turn occurred.

1.4 Finding

My finding is that the MH370 route began with a southward turn off the extended military radar track at 18:27 UTC, transitioning into a great circle path directly over Maimun Saleh Airport on Weh Island at an initial bearing of 192.3 degrees, and ending at the 00:19 UTC handshake arc at geodetic coordinates (40.24S, 83.53E).

2 Model Assumptions

2.1 Number of Turns

I began my analysis by analyzing routes having one or two turns after the last radar contact at 18:22 UTC. I found that only one southward turn is needed to match the satellite data extremely well. All the calculations presented in this analysis utilize only one turn. The BFO data are also indicative of a single final turn southward at a point in time between 18:25 and 18:40 UTC.

2.2 List of Assumptions

Here is a list of the assumptions I have made for this analysis:

1. The BTO, BFO, and satellite position data given at the seven handshake arc times in the ATSB Report and in the MH370 Data Communication Logs are correct.
2. The military radar data are correct, and the last known military radar contact occurred at 18:22 UTC.
3. From that final radar location at 18:22 UTC until a point in time past 22:41 UTC, the plane flew :
   a. at constant altitude (FL350 = 35,000 feet),
   b. with a single southward turn between 18:25 and 18:40 UTC,
   c. under autopilot control of Lateral Navigation in one of five modes:
      i. constant true track, or
      ii. constant magnetic track, or
      iii. constant true heading, or
      iv. constant magnetic heading, or
      v. great circle, and
   d. with engine thrust controlled by one of three means:
      i. the engine throttles were manually set and then left unchanged, or
      ii. the auto-throttle was engaged to maintain a previously entered Mach number, or
      iii. the auto-throttle was set to maintain a speed determined by the Flight Management System using the Cost Index parameter.

3 Implications of Assumptions

3.1 Satellite Data

Based on Assumption 1 above (accurate satellite data), I have utilized the BTO and BFO data in order to generate and to evaluate the entire range of possible southern routes. The process of route synthesis I used provided direct determination of aircraft positions at the seven handshake event times identified by the ATSB in order to assure close matches to the handshake arc radii that I calculated from the BTO values. In addition, I have evaluated the BFO values of the most-promising routes to determine if the measured and predicted BFO values are consistent within the expected noise level. Note that I have not used the BFO data directly in the route fitting process. Instead, I have used the combination of handshake arc radii (from the BTO data), the military radar data, and the variations in true air speed along the route to synthesize routes at all southerly course settings for all Lateral Navigation modes. Then I used the BFO data as a check to assure validity of the selected route.

3.2 Military Radar Data

Assumption 2 (that the military radar positions and times are correct) is critical in limiting the possible paths of 9M-MRO between 18:22 and 19:41 UTC. Indeed, the uncertainty in the MH370 end point is primarily caused by the lack of an accurately known position after the final southward turn. The key to reducing the length of the 7th arc that must be searched is to apply other constraints to eliminate implausible routes.

3.3 Other Assumptions

Assumption 3.a (constant altitude) is utilized because there is no direct evidence of any descent until the aircraft ran out of fuel. FL350 (35,000 feet) was the altitude last reported by the ACARS, and it also seems likely that no change in altitude was made by the flight crew after the final turn (although there is indirect evidence for an altitude change during the turn).

Assumption 3.c simply requires that all possible Lateral Navigation settings in the five modes be evaluated.

3.4 Engine Thrust Control

Assumption 3.d (either manual throttle or auto-throttle) implies essentially constant engine thrust and therefore very stable air speed.

3.4.1 Manual Throttle

With initial manual setting of engine thrust levers (Assumption 3.d.i above), auto-throttle is disabled, but the aircraft air speed will still be steady with perhaps only a very small and slow drift as ambient air conditions (such as temperature) change along the flight path.

3.4.2 Auto-Throttle

With auto-throttle control of Mach number enabled (Assumptions 3.d.ii and 3.d.iii) the air speed will normally be very steady. How steady the air speed is will depend, for instance, on how much the ambient temperature varies along the route. The Mach number is not directly affected by the air temperature. However, the effect of say, colder air (at increasing distance from the equator) is to increase the engine thrust. When the auto-throttle computer compensates by reducing its power setting to keep the thrust (and Mach number) constant, the aircraft will slow down a bit and the true air speed will be slightly reduced. For a 1 K drift downward in ambient temperature, the true air speed drift will be at most 1 knot downward. There is a partially compensating effect in that the colder air is denser and the air drag on the aircraft will increase, requiring a slightly higher thrust setting to maintain Mach number. The net effect is that the true air speed will be very stable along the MH370 route.

The air temperature at 35,000 feet altitude over the portions of the routes of interest here (from IGARI until 22:41 UTC) is extremely stable. The NOAA Global Data Assimilation System archived temperature data for midnight on March 7, 2014 at 250 hPa pressure is 232 K +/- 1 K from the diversion near IGARI until well past the 21:41 UTC arc crossing. Nearing the 22:41 UTC arc the temperature falls to 228 K, and one would expect the true air speed to drift a knot or two lower for that flight segment.

It is also worth noting that virtually all fitted routes show a decreasing true air speed for the last two (farthest south) flight segments ending at 00:11 and at 00:19 UTC. It is highly likely that the majority of this decrease is due to fuel exhaustion, although a small portion (up to ~7 knots) of the true air speed decrease at 00:11 may be due to this air temperature effect.

With the engine thrust control set using a Cost Index (Assumption 3.d.iii), the variability of the air speed will be the same as the previous case of auto-throttle control of Mach number. Once again, only very small and slow variations in true air speed are expected to occur.

3.5 **Air Speed Control**

I believe a reasonable upper limit is < 10 knots peak-to-peak variation for any of the thrust control modes in Assumption 3.d. For a relatively small number of flight segments, this corresponds to an upper limit of ~3 knots standard deviation, and the expectation is that it should be even lower.

This expectation of stable true air speed (= "steady" true air speed) provides the additional means of filtering out those potential routes that otherwise are consistent with the BTO and BFO data. This principle is the essence of my analysis – the exclusion of routes that have "unsteady" true air speed.

4 Flight Route Model

4.1 **EXCEL Model**

I created a flight route modeling program using Microsoft EXCEL. The starting point is the final radar position. It fits route segments so that three conditions are satisfied simultaneously. These conditions are as follows:

1) The locations of the aircraft at the times of the seven handshakes are within 1.5 nautical miles (NM) RMS radial error of the arc radii derived from the BTO satellite data. Table 9-7 in the Appendix shows my method of calculating arc radii and typical results. The 1.5 NM error limit I used is equivalent to ~13 μsec in BTO when averaged over all rings. This is smaller than the 26 μsec 1-sigma error assigned by the ATSB to the BTO values based on the variability at KLIA prior to takeoff. I suspect that the in-flight repeatability of the BTO may actually be better than it was during the airport measurements. The reason for this is that, at the airport, the BTO data may have been degraded by multi-path propagation effects (reflections) from nearby buildings and other aircraft. These effects will not occur once the aircraft has taken off and is isolated from nearby reflecting bodies. In any case, allowing only 13 μsec RMS BTO error still allows fits demonstrating a small fraction of one knot standard deviation of true air speed (which is really quite amazing in terms of the precision!). Increasing the BTO error to 26 μsec does not significantly change my best-fit route solutions.

2) The track/heading setting of the Lateral Navigation system is accurately maintained so that the values during all flight segments between pairs of handshake arcs are within a standard deviation of 0.10 degrees of the commanded values. Note that this is not the same as saying the lateral navigation always maintained its course within 0.10 degrees, just that the average course measured over an hour or so between handshake arcs was within 0.10 degrees RMS error of the desired course.
3) The standard deviation about the (weighted) mean of the true air speed for the route segments between the last radar point at 18:22 UTC and the 22:41 UTC handshake arc "crossing" is minimized. I have selected the 22:41 UTC handshake as the last point for inclusion in the "steady true air speed" calculations for two reasons. First, it is possible (even likely) that one engine ran out of fuel shortly before 00:11 UTC, causing a decay in air speed. Second, my analysis of the true air speed shows that there was indeed a reduction in true air speed for the 22:41 to 00:11 UTC leg followed by a further reduction in true air speed between 00:11 and 00:19 UTC.

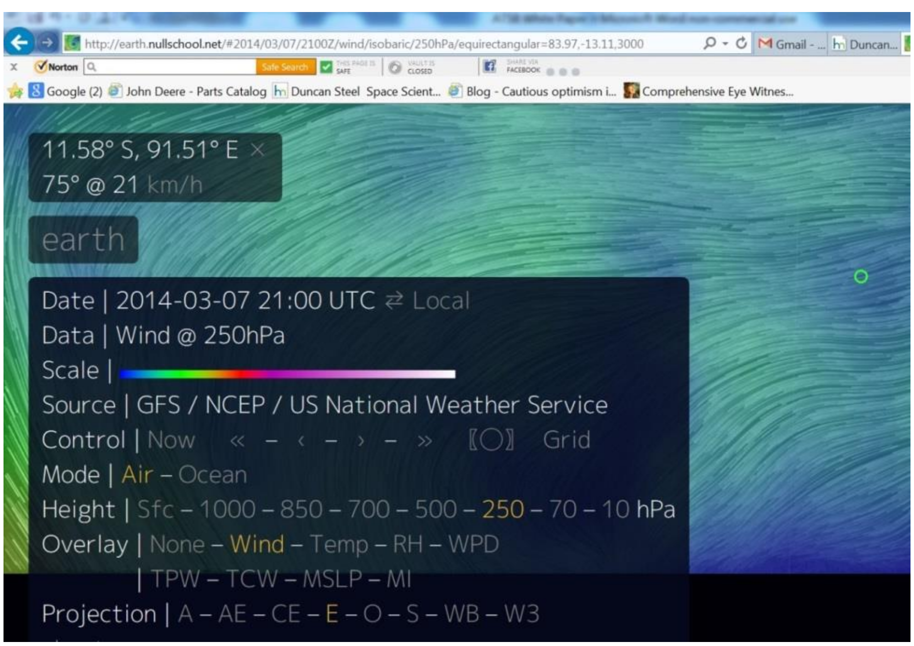
4.2 **Wind**

Winds at the aircraft altitude affect the relationship between ground speed and air speed. The EXCEL route is based on a sequence of latitude/longitude positions and epochs. Knowing range and elapsed time for each segment, one can calculate the average ground speed. Knowing the wind speed and direction, one can then calculate the true air speed and heading for each segment. Thus having a map of wind speed and direction at the location, flight time, and correct altitude is necessary to evaluate the steadiness of true air speed. Fortunately (and somewhat surprisingly), such data are available. Figure 4-1 shows wind data selected for March 7, 2014 at 2100 UTC and for an effective altitude of 250 hPa, which is the closest setting equivalent to FL350. Here is a link to the wind data:

http://earth.nullschool.net/#2014/03/07/2100Z/wind/isobaric/250hPa/equirectangular=93.05,-37.69,2048

I wish to thank Mr. Brian Anderson who provided me with a table of these wind data at whole degree increments for the region of interest. My EXCEL program automatically selects the entry at the nearest data table location. In order to improve the accuracy, I used the vector average of the wind speed and direction for the two end points of each flight segment.

Figure 4-1 Wind Data for MH370 Flight Conditions



The wind corrections are quite effective, as evidenced by the fact that routes with $>$ 5 knots standard deviation of ground speed show $<$ 1 knot standard deviation of true air speed.

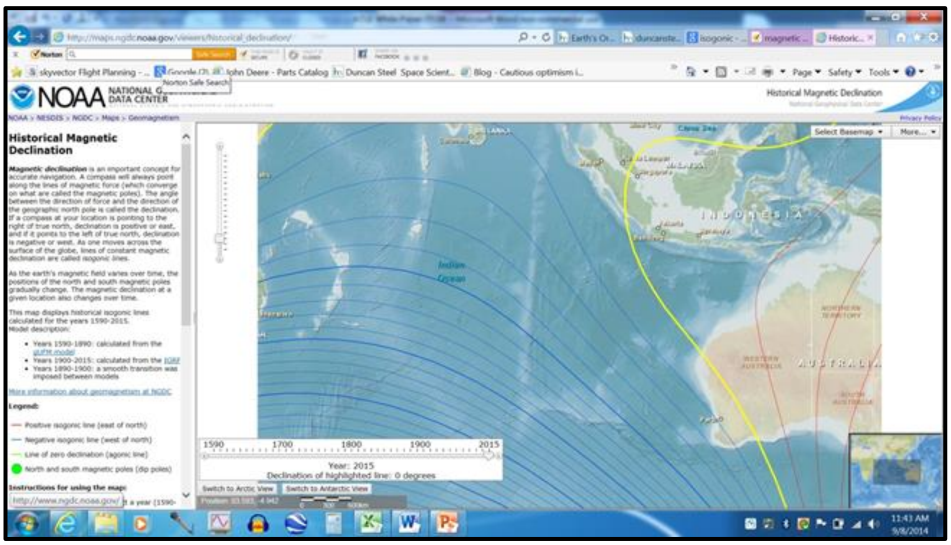
4.3 <u>Magnetic Declination</u>

Knowledge of the magnetic declination along a flight route is necessary in order to generate the magnetic track and magnetic heading routes. NOAA provides such data here:

http://maps.ngdc.noaa.gov/viewers/historical_declination/

A screen shot is shown in Figure 4-2. I used the 2015 data since it most closely matches the 2014 MH370 flight date.

Figure 4-2 Magnetic Declination Chart



This internet site only allows one to select a latitude/longitude position and view the isogonic lines. Initial attempts to fit routes by manually entering magnetic declination into the EXCEL flight model at each event location were very time consuming. This prevented detailed assessment of the routes with magnetic reference.

To streamline the magnetic route fitting process, I developed a closed-form solution. I first created a data base by manually interpolating between isogonics at every whole degree of latitude and longitude over the area of interest (15N to 45S, 80E to 110E). Then I fit an equation to that data table with the following form:

(Eq. 1)
$$MagDec = mdcoef0 + mdcoef1*ATAN2(lat+64,137-lon)+ mdcoef2*(ATAN2(lat+64,137-lon))^2 + mdcoef3*SQRT((lat+64)^2+(137-lon)^2) + mdcoef4*(SQRT((lat+64)^2+(137-lon)^2))^2 + mdcoef5*ATAN2(lat+64,137-lon)*SQRT((lat+64)^2+(137-lon)^2) + mdcoef6*ATAN2(lat+64,137-lon)*SQRT((lat+64)^2+(137-lon)^2)^2 + mdcoef7*(SQRT((lat+64)^2+(137-lon)^2))*(ATAN2(lat+64,137-lon))^2 +mdcoef8*LOG(ATAN2(lat+64,137-lon)).$$

Table 4-1 lists the coefficients I derived by fitting Equation (1) to the interpolated data set using the least-squares method.

The residual errors of the approximate Equation (1) above are 0.4 degrees RMS and 1.4 degrees peak. This is sufficiently accurate for my route analysis, and having a closed-form solution allowed magnetic routes to be computer-generated in reasonably short computer-processing times. To improve the accuracy, I used the average of the magnetic declination values at the two end points and at the midpoint of each flight segment. I used the approximate but continuous Equation (1) above rather than a discontinuous look-up

table at one degree spacing because a continuous function generally allows optimization algorithms to converge more rapidly.

Table 4-1 Magnetic Declination Equation Coefficients

Magnetic Declination Coefficients	
Name	**Value**
mdcoef0	1.05515E+02
mdcoef1	-1.99560E+02
mdcoef2	1.14834E+02
mdcoef3	-5.94556E-01
mdcoef4	6.15283E-03
mdcoef5	6.59800E-01
mdcoef6	-7.94367E-03
mdcoef7	5.31207E-02
mdcoef8	9.21705E+01

4.4 Turn Model

Inspection of the radar tracks shown in Figure 2 of the ATSB report demonstrates that the aircraft turns near IGARI and Penang were made with a turning radius of approximately 6-10 NM. I created a turning model in EXCEL that uses a 1.0 degree / second rate turn. This is a 1/3 standard rate turn, and it is consistent with the 25 degree maximum bank angle selection using the Mode Control Panel in a Boeing 777-200ER aircraft. At 500 knots, the turning radius is ~9 NM.

The EXCEL turn model receives inputs of initial position, initial bearing, final bearing, and ground speed. It outputs a turn duration, the length of the track flown during the turn, and a final aircraft position for the end of the turn.

Having this turn model is essential to including the final turn segment in the air speed calculations. In the route tables in the Appendix, the distances traveled and the elapsed times for the turn segment come from the turn model, not from a straight-path calculation between the turn end points.

4.5 Earth Model

I used an ellipsoidal Earth model (with WGS-84 radii), and I have corrected all the range calculations, including the handshake arc radii, for the latitude dependency of the Earth's radius. I integrated the average Earth radius from the sub-satellite point to the latitude of the aircraft at the arc crossing to improve the accuracy. I developed a simple polynomial fit to this function that allowed it to be automatically updated as the route fitting program was running.

I used geocentric latitude in my handshake arc model for computational convenience. I have given the event locations of the Maimun Saleh Airport route in both geocentric and geodetic coordinates to allow direct comparison with other modeling results and search maps.

4.6 Route Fitting Methods

4.6.1 Nonlinear Solver

I used an EXCEL spreadsheet program I wrote to synthesize MH370 routes. The starting point is the aircraft position and bearing at the last radar contact at 18:22 UTC. The end point is the 00:19 UTC handshake arc intersection. The program adjusts various parameters (such as range, bearing, and time and speed of the final turn) that define the route in such a way that a weighted sum of the standard deviations of the three parameters described previously in Section 4.1 (radial errors, course errors, and true air speed variations) was minimized. I used the GRG Nonlinear Solver function in EXCEL to fit the optimum route parameters.

The program is actually six distinct sets (= 6 files) of worksheets, one file for each of the five Lateral Navigation modes and one file for linking those results into integrated graphics. Examples of best-fit route parameters and handshake arc calculations for each of the five Lateral Navigation modes are shown in the Appendix in Tables 9-1 to 9-5.

For each Lateral Navigation setting, I used a two-step fitting process. First, I used weighting factors that caused the bearing errors and the handshake arc radial position errors to both become very small (i.e., ~zero) without minimizing the air speed errors. Second, I changed the weighting factors in the optimization function and employed the two upper limits described above [(a) the RMS radial error from the seven handshake arcs was < 1.50 NM, and (b) the RMS course error was < 0.10 degrees] so that the standard deviation of the true air speed was minimized from 18:22 to 22:41 UTC. In computing the mean and standard deviations of the true air speed, I weighted the values for each flight segment by the range (length in NM) of that segment. This weighting is necessary to avoid having short-leg errors dominate the statistical results.

4.6.2 Lateral Navigation Modes

Four of the five Lateral Navigation modes employ settings entered using the Mode Control Panel in one degree increments. Either Track or Heading can be selected directly on the Mode Control Panel. In addition, either Normal Reference (which is the default magnetic reference) or True Reference can be selected via a switch in the Captain's inboard display panel. So we have a total of four possible Lateral Navigation modes set using the Mode Control Panel.

An additional Lateral Navigation mode, employing great circle routes, is available using the Flight Management System. In this mode a waypoint, or a sequence of waypoints (called an airway), is selected. However, if that waypoint location is later passed by the aircraft with no additional inputs, by default the Flight Management System continues to follow that same great circle path. In this great circle navigation case, the granularity of the initial bearing of the great circle path can be quite small (much less than one degree). For the purposes of my analysis, I began by evaluating great circle routes with integer degree initial bearings at the completion of the final turn. That provides sufficiently fine resolution to discern if there are any great circle initial bearings for which the true air speed is very steady, implying that those routes are therefore consistent with autopilot control and should be considered plausible in refining the search area.

As described below, I also evaluated great circle routes passing "through" nearby airports and aviation waypoints.

4.6.2.1 True Track

For the Lateral Navigation mode of constant true track, I assumed the aircraft initially continued on the same bearing that was held between the 18:05 and the 18:22 UTC military radar contacts (~289 degrees). I also assumed that one major turn occurred between the 18:25 UTC handshake and the 19:41 UTC handshake. I further assumed the autopilot was set to follow a true track upon completion of the final turn.

I fit 18 different true track bearings between 172 and 197 degrees, with one degree spacing near the steadiest speeds. All parameters for the best-fit route at 192 degrees are shown in Table 9-1 in the Appendix. It is apparent that the wind corrections applied to convert ground speed to true air speed are effective. In this case the ground speed variation is ~6 knots, but the true air speed standard deviation is less than 1 knot.

I found a true track route (at 192 degrees) that meets all the constraints listed in Section 4.1. However, it is a less-likely solution than a great circle route, all things else being equal. The reason for this is that the flight crew normally navigates using waypoints and standard airways in a great circle mode through the Flight Management System. The Mode Control Panel is used primarily when taking off and landing. In

addition, in order to select True Reference, rather than Normal (magnetic) Reference, a switch must be thrown that is normally not used.

4.6.2.2 Magnetic Track

The assumptions for the constant magnetic track case are identical to those described above for the constant true track case. I fit 22 different magnetic track bearings between 147 and 197 degrees, with one degree spacing near the steadiest speeds. All parameters for the best-fit route at 188 degrees are shown in Table 9-2 in the Appendix.

No magnetic track route has a very steady true air speed; the best case is ~4 knots standard deviation.

4.6.2.3 True Heading

The assumptions for the constant true heading case are identical to those listed above for the constant true track case. I fit 16 different true headings between 150 and 194 degrees, with one degree spacing near the steadiest speeds. All parameters for the best-fit route at 185 degrees are shown in Table 9-3 in the Appendix. No true heading route has very steady true air speed. The best case has ~5 knot true air speed standard deviation.

It seems unlikely that the flight crew would choose to fly in a heading mode, and indeed, none of these true heading routes proved promising.

4.6.2.4 Magnetic Heading

For the Lateral Navigation mode of constant magnetic heading, I assumed a similar scenario to the constant true heading case and that the autopilot was set to follow a magnetic heading on the completion of that turn.

I fit and evaluated 26 magnetic headings between 137 and 194 degrees. The best fit is at 184 degrees, and the parameters for the best-fit route are shown in Table 9-4 in the Appendix. No magnetic heading route has very steady true air speed, and the lowest true air speed standard deviation is ~6 knots, which is not plausible. It also seems unlikely that the flight crew would choose to fly in magnetic heading mode, and indeed, none of these routes proved satisfactory.

4.6.2.5 Great Circle

The fifth Lateral Navigation mode is great circles using the Flight Management System. The assumptions in this case are the same as described previously for the four Mode Control Panel methods.

First I tried setting the initial bearing in integer degrees over the range of 143 to 195 degrees. I fit 20 initial bearings and found that the 190-193 degree range produced routes with less than one knot true air speed standard deviation.

Next I found the initial bearing that produced the steadiest true air speed (less than 1 knot), and that was at 192.1 degrees. Table 9-5 in the Appendix shows those route parameters.

Inspection of the aviation waypoints southward of the aircraft location at the onset of the final turn produced a relatively short list: SANOB, IGEBO, and ANSAX. In addition, there is an airport at Banda Aceh, Indonesia called Sultan Iskandar Muda Airport (BTJ / WITT) with a 10,000 foot asphalt runway. That would certainly be a logical place to go with a somewhat disabled aircraft after not being able to land at Penang International Airport (VPG). Perhaps the flight crew judged that attempting an emergency landing at an active airport (Kota Bharu or Penang or Kuala Lumpur) would be too risky with no operational radar transponder or radio. There would certainly be some risk of a collision with another aircraft either in the air or on the runway. In any case, the aircraft flew northwest, perhaps heading toward Banda Aceh with the nearest long runway and no significant danger of collision since that airport is normally closed at that time of the night.

I tried fitting numerous routes, including the following:

a) turn and fly to SANOB,
b) turn and fly to BAC,
c) turn and fly to SANOB and then fly to BAC,
d) turn and fly to ANSAX,
e) turn and fly to ANSAX and then fly to BAC, and
f) turn and fly to IGEBO.

None of these routes proved entirely satisfactory. The routes through Banda Aceh could be fit with < 1 knot true air speed error, but the turn times were such that the turn was completed within a few seconds after 18:27 UTC. That turn time is inconsistent with the BFO data, which indicate that the turn was still underway at 18:28:06 and also at 18:28:15 UTC.

It became clear that the best-fit (steady true air speed) great circle route went west of Banda Aceh (BAC) by ~10 NM. The turn for this route started at ~18:26:54 and ended at ~18:28:30 UTC. This turn time is consistent with the BFO data showing a turn in progress at ~18:28:10 UTC. That agreement in turn time with the BFO data provides a great deal of confidence that this route is the correct one. The next question is which waypoint or airport was used to set the great circle route?

4.6.2.6 Maimun Saleh Airport

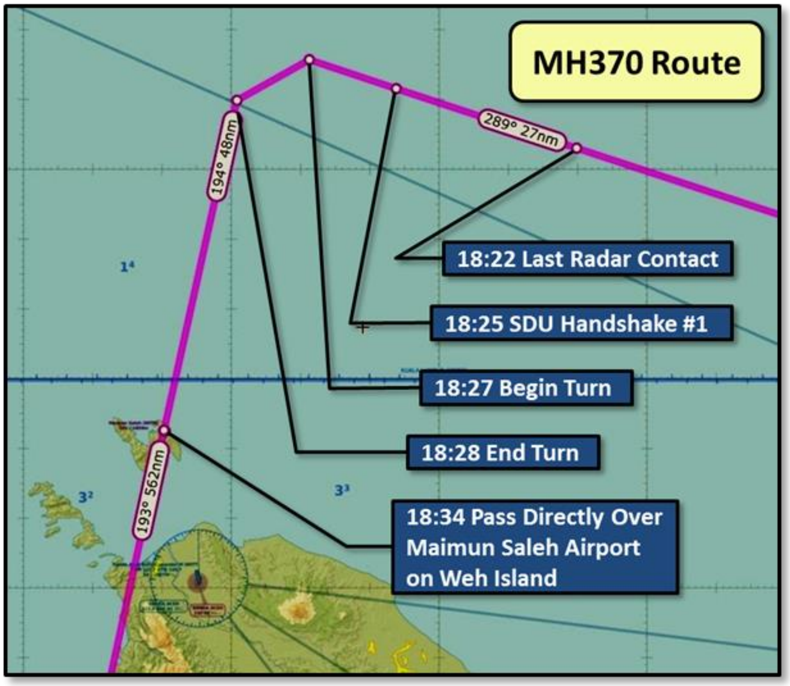
Further inspection of aeronautical charts proved fruitful. There is an airport on Weh Island about 25 NM north of Banda Aceh. It is the Maimun Saleh Airport (IATA: SBG / ICAO: WITN), located in Sabang, Pulau Weh, Indonesia. It has a 6,000 foot paved and lighted runway. It is a general aviation airport with no scheduled airline service. With minimal air traffic, collision risk is also minimal. Furthermore, it is closer to 9M-MRO than Banda Aceh when the flight crew restores electrical power to some of the onboard equipment (including the Satellite Data Unit) shortly before the 18:25 – 18:28 UTC Satellite Data Unit transmissions.

It seems quite fortunate in hindsight that the Satellite Data Unit is communicating just when the aircraft makes a turn. This may not be coincidental. It would seem plausible that the time for making a (third?) landing attempt was growing short for several reasons, not the least of which was that the aircraft was nearly abreast of the last airport(s) available for an emergency landing before heading out to open sea. Therefore it appears that actions were taken, and two of those steps appear to be first restoring electrical power (thus rebooting the Satellite Data Unit) and then turning the aircraft to the nearest airport (Maimun Saleh).

The route which best fits the satellite data with steady true air speed passes directly over the Maimun Saleh Airport (without descending). I infer that the flight crew made a purposeful turn and headed directly to it. This route is in the process of turning between 18:27:00 and 18:28:36 UTC, and it has a true air speed standard deviation of less than 1 knot. Details of this route are shown later in this report in Tables 6-2, 6-3, and 9-6. The initial bearing to Maimun Saleh Airport after the turn is 192.36 degrees, and the approximate time of passage over that airport is 18:34:23 UTC on March 7, 2014.

Figure 4-3 shows the most-likely route from the last radar contact through the turn and passing over Maimun Saleh Airport as displayed by the "flight plan" software on SkyVector.com. The notes on the map show the times and locations of 9M-MRO from the last radar contact at 18:22 UTC until its passage over Maimun Saleh Airport at 18:34 UTC.

Figure 4-3 MH370 Route Over Maimun Saleh Airport



In my judgment the Maimun Saleh Airport route is the route flown by the MH370 flight crew for the following reasons:

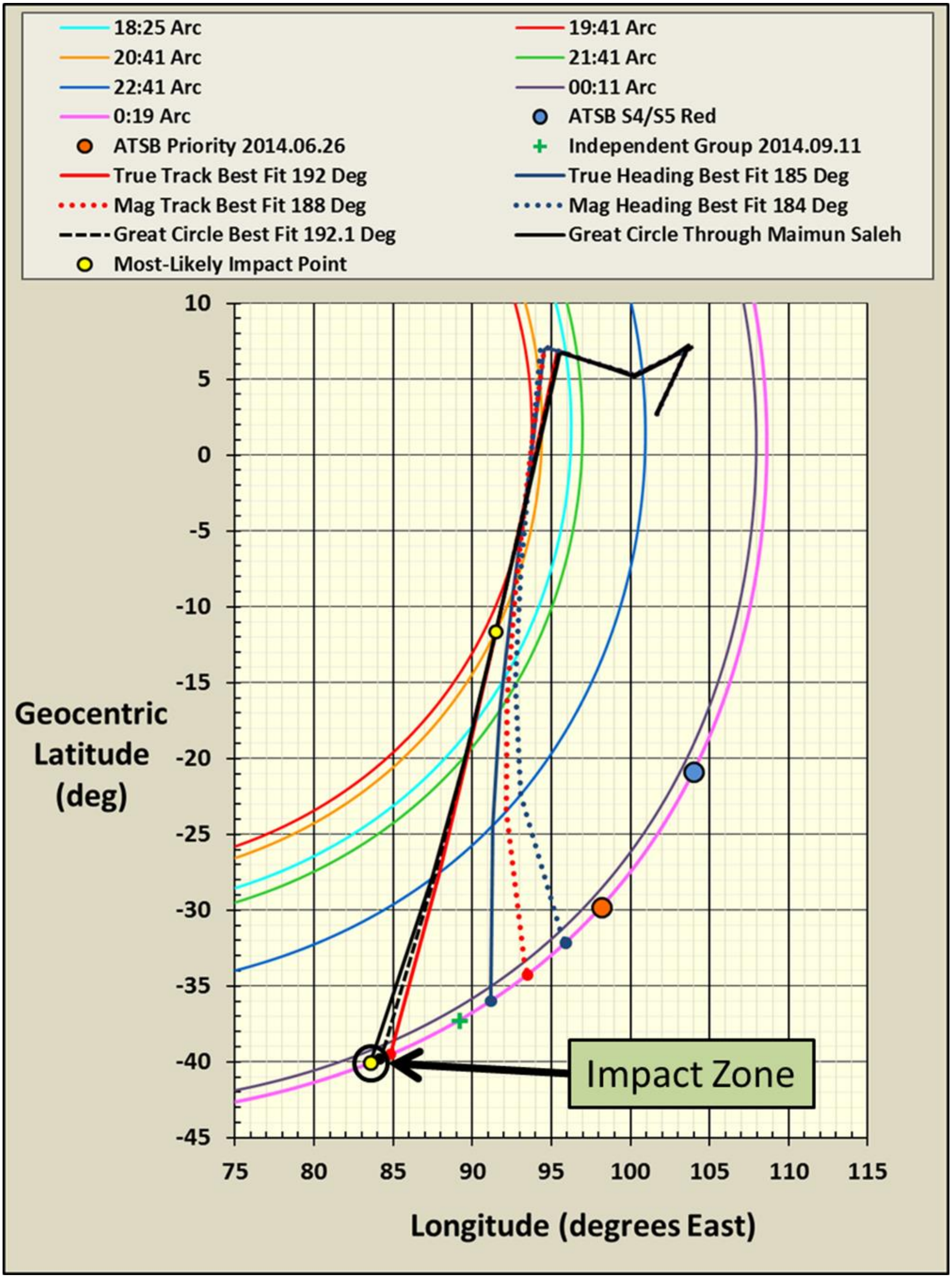
1. **it is a plausible, nearby destination (the nearest airport),**
2. **it is a great circle route using the Flight Management System (the normal navigation mode),**
3. **it is an extremely simple route, requiring only one turn and no speed changes,**
4. **it matches the handshake arcs derived from the BTO data very closely (< 1.5 NM RMS error),**
5. **it has extremely steady true air speed as expected (~ 1 knot stability),**
6. **it matches the average true air speed derived from the post-diversion radar track (within 1 %),**
7. **it matches the BFO data for the entire flight (~ 5 Hz residual RMS error), and**
8. **it matches the southward turn completion time from the BFO data at 18:28:46 UTC (within 20 seconds).**

5 Summary of All Route Fitting Results

5.1 Best Fit Routes

The best-fit routes for the five lateral navigation modes are shown in Figure 5-1. In this case "best" means that it has the "steadiest" true air speed.

Figure 5-1 Best-Fit Routes for Five Lateral Navigation Modes



The blue "dot" along the 7th handshake arc at 00:19 UTC is the center of the previous ATSB S4/S5 Red search area. The orange "dot" is the center of the current ATSB Priority Search Area. The seven handshake arcs are shown in the figure. You will note that my seventh arc at 00:19 UTC appears to match closely with the ATSB arc location indicated by the ATSB search area centers.

All MH370 routes are identical until the last radar contact at 18:22 UTC. After that time they begin to diverge. You will note that all routes appear to cross the 19:41 UTC arc in the vicinity of -2 to -3 degrees, where they are nearly tangent to the arc.

Moving southwesterly along the 7th arc, we first come to the end point of the blue dotted route, which is the magnetic heading best fit at 184 degrees. Next along the arc is the red dotted route – the magnetic track at 188 degrees. Next is the blue solid line for the true heading at 185 degrees.

The last group has three routes which are nearly identical. They include the red line for the true track at 192 degrees, the dashed black line for the 192.1 degree great circle route, and finally the solid black line for the Maimun Saleh Airport great circle route at 192.3 degrees initial bearing. As mentioned previously, this route appears to be the most plausible (="most-likely") and matches the satellite data better than all other routes. Its end point on the 7th arc is marked by a yellow "dot." This is the most-likely 9M-MRO impact point.

5.2 **True Air Speed Variation**

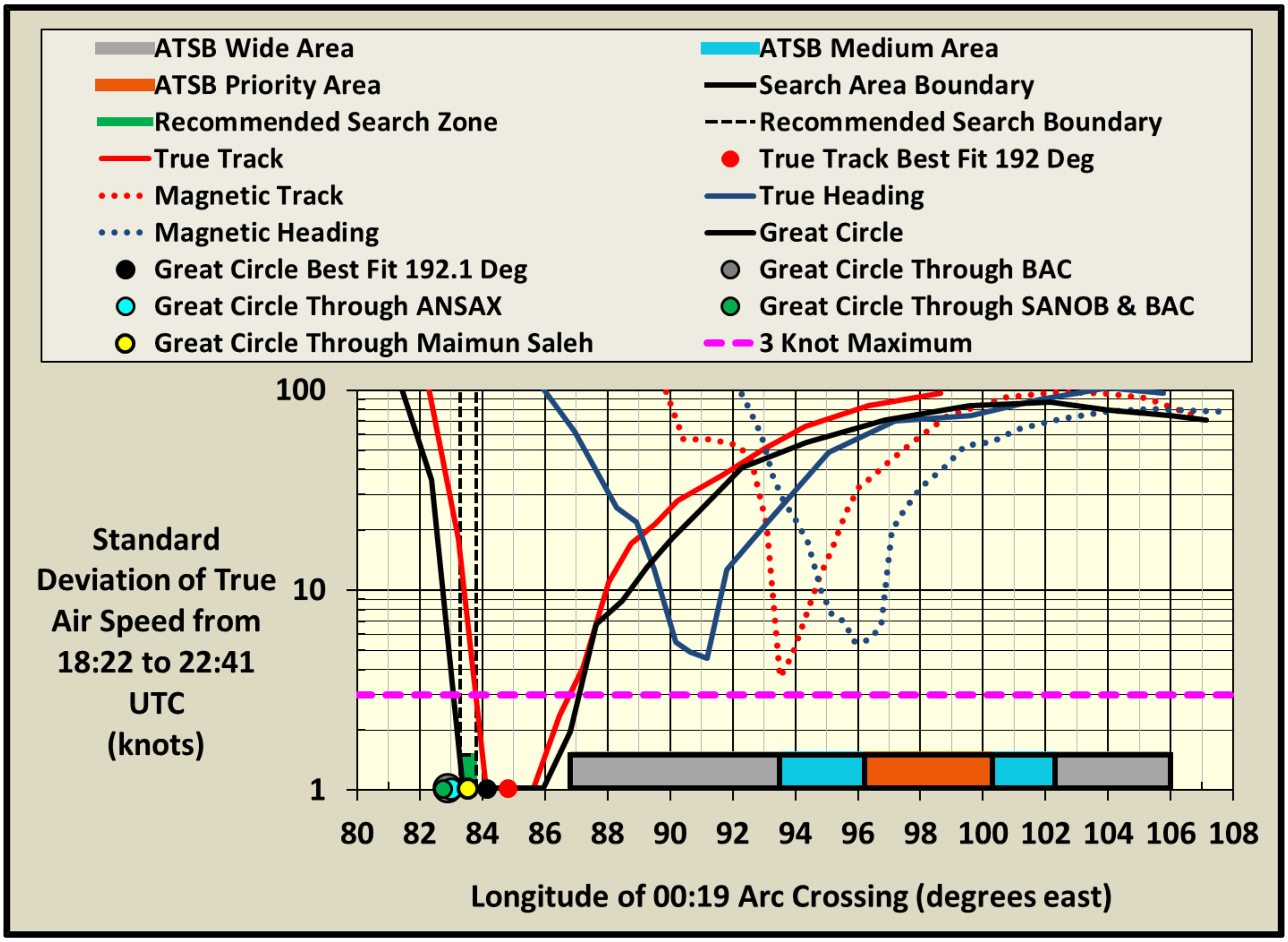
Figure 5-2 shows the standard deviations of the true air speeds for all the routes evaluated. This graph represents the majority of the effort involved in this analysis. These curves should be interpreted as being "frontiers," or lower limits to the true value for MH370. The computer fits will produce somewhat smaller one-sigma true air speed values than is actually the case simply because of the number of variables fitted and the presence of noise in the BTO data.

The horizontal axis of Figure 5-2 is the longitude of the end point of the route on the 7th arc (at 00:19 UTC). It is convenient to use this single longitudinal dimension to locate the impact points, since there is no controversy about the plane impacting very near the 7th arc. This also allows one to identify routes based on the single most important feature – the longitude of the end point. I have followed this convention in many of the graphs to follow. I also note that the longitude coordinate is identical in value in both geocentric and in geodetic coordinate systems.

Looking down at the bottom of Figure 5-2, you will note that I have marked the three ATSB Search Areas – Wide in gray, Medium in blue, and Priority in orange (following the ATSB Report convention). I have also shown a recommended search zone in green. It is derived in Section 7 after consideration of all route selection criteria.

There are five curves in Figure 5-2 – one for each of the Lateral Navigation control modes. The vertical axis in Figure 5-2 is the standard deviation of the true air speed on a logarithmic scale from 1 to 100 knots. The curves for each Lateral Navigation mode display very large speed variations on both the left and right ends (~100 knots). In between there is a dip to a minimum value. As mentioned previously, I estimated an upper limit for the expected speed variation equal to 3 knots, and this is shown as the pink dashed line running horizontally across the graph in Figure 5-2.

Figure 5-2 Standard Deviation of True Air Speed for All Routes



Further inspection of Figure 5-2 shows that two of the Lateral Navigation modes have speed variations of ~1 knot (or better). These modes are the true track best fits (the red line through the red dot) and the great circle best fits (the black line through the black dot).

The mathematical residual errors in those fits marked as 1 knot in Figure 5-2 are actually much less than 1 knot. However, this is a mathematical artifice that results from the number of variables used in the fitting process, and I do not expect the actual speed variations of 9M-MRO to be noticeably smaller than ~1 knot. Therefore I have "clipped" the fitted results below 1 knot and assigned them all a 1 knot value, indicating essentially "perfect" fits.

You will also note four additional "dots" in Figure 5-2 having 1 knot speed stability. The gray dot is the great circle route through Banda Aceh Airport (BAC). The blue dot is the great circle route through ANSAX. The green dot is the great circle route through SANOB and then to BAC. As discussed previously, for other reasons these routes are less likely than the one through Maimun Saleh Airport, shown as a yellow dot. You will also see at the bottom of Figure 5-2 that all satisfactory routes end within approximately 83-86 degrees longitude, and none fall within any of the ATSB's current search areas.

5.3 <u>Average True Air Speed</u>

The next consideration is the average true air speed. The specified maximum speed of the 777-200ER aircraft is Mach 0.89, or ~513 knots true air speed at 35,000 feet altitude. Typical cruising speed is Mach 0.83-0.84 (478-484 knots).

5.3.1 Pre-Diversion True Air Speed

After reaching 35,000 feet altitude, but prior to the emergency in-flight diversion, 9M-MRO was flying at a ground speed of ~471 knots at a bearing of 40 degrees based on the last secondary radar data at 17:22 UTC. This was during the time when 9M-MRO was turning right from the IGARI bearing of 25 degrees to the BITOD bearing of 59 degrees. At 471 knots ground speed at 40 degrees bearing, and with a 9 knot headwind from 68 degrees, the true air speed from the last ACARS data was 479 knots (Mach 0.83). Thus we see excellent agreement between the actual 9M-MRO true air speed prior to diversion (Mach 0.83) and the expected typical cruise speed (Mach 0.83-0.84). As shown in Table 5-1 below, based on the radar track the average true air speed from the last ACARS data to the diversion turn-around was 478 knots (again Mach 0.83).

Table 5-1 Radar Track Analysis

Time	Event or Location	Aircraft Position				Leg From Previous Position					Wind at Aircraft Position	
									Using Average of Winds at Leg Ends			
(UTC)		Geodetic Latitude (deg N)	Geocentric Latitude (deg N)	Longitude (deg E)	Altitude (ft)	Range (NM)	Initial Bearing (deg)	Average Ground Speed (knots)	True Air Speed (knots)	True Heading (deg)	Direction (deg)	Speed (knots)
2014/3/7 16:41:00	Takeoff from Kuala Lumpur Int. Airport (VKL)	2.727	2.709	101.710	69						69	18.2
2014/3/7 16:44:00	Turn to IGARI	2.860	2.841	101.670	2,600	8.3	343.2	166	168	349.3	69	18.2
2014/3/7 17:06:43	Last ACARS Data	5.290	5.255	102.800	35,000	160.1	25.0	423	435	26.5	67	14.6
2014/3/7 17:20:39	Arrive IGARI	6.937	6.891	103.585	35,000	108.9	25.5	469	478	26.4	68	8.8
2014/3/7 17:21:37	Last Secondary Radar Data	7.020	6.974	103.680	35,000	7.5	48.7	470	478	49.1	68	8.8
2014/3/7 17:22:20	Begin Diversion Turn-Around	7.070	7.023	103.760	35,000	5.6	58.0	469	478	58.2	68	8.8
2014/3/7 17:24:39	End Diversion Turn-Around	7.260	7.212	103.680	35,000	19.3	337.2	500	500	338.2	68	8.8
2014/3/7 17:36:38	Arrive Kota Bharu Airport (GOLUD)	6.285	6.243	102.278	35,000	102.0	235.2	511	500	235.1	54	12.5
2014/3/7 17:52:37	Arrive Penang International Airport (VPG)	5.180	5.146	100.260	35,000	137.5	241.3	516	500	241.6	51	19.9
2014/3/7 18:02:33	Military Radar - Pulau Perak	5.660	5.622	98.940	35,000	84.0	289.9	508	500	291.6	45	14.0
2014/3/7 18:22:12	Military Radar - Final Position	6.550	6.507	96.330	35,000	164.8	288.8	503	499	289.7	50	3.0

Item	Average Ground Speed (knots)	Weighted Average True Air Speed (knots)	Estimated 1-Sigma Uncertainty in Each End Point		Total Range (NM)	Total Elapsed Time (hh:mm:ss)	Estimated Standard Deviation of Average True Air Speed (knots)
			Range (NM)	Time (s)			
From "Last ACARS Data" To "Begin Diversion Turn-Around"	469	478	2	5	122.1	0:15:37	12
From "Military Radar - Palau Perak" To "Military Radar -Final Position"	503	499	2	5	164.8	0:19:39	9
From "Begin Diversion Turn-Around" To "Military Radar -Final Position"	509	500	2	5	507.7	0:59:52	3

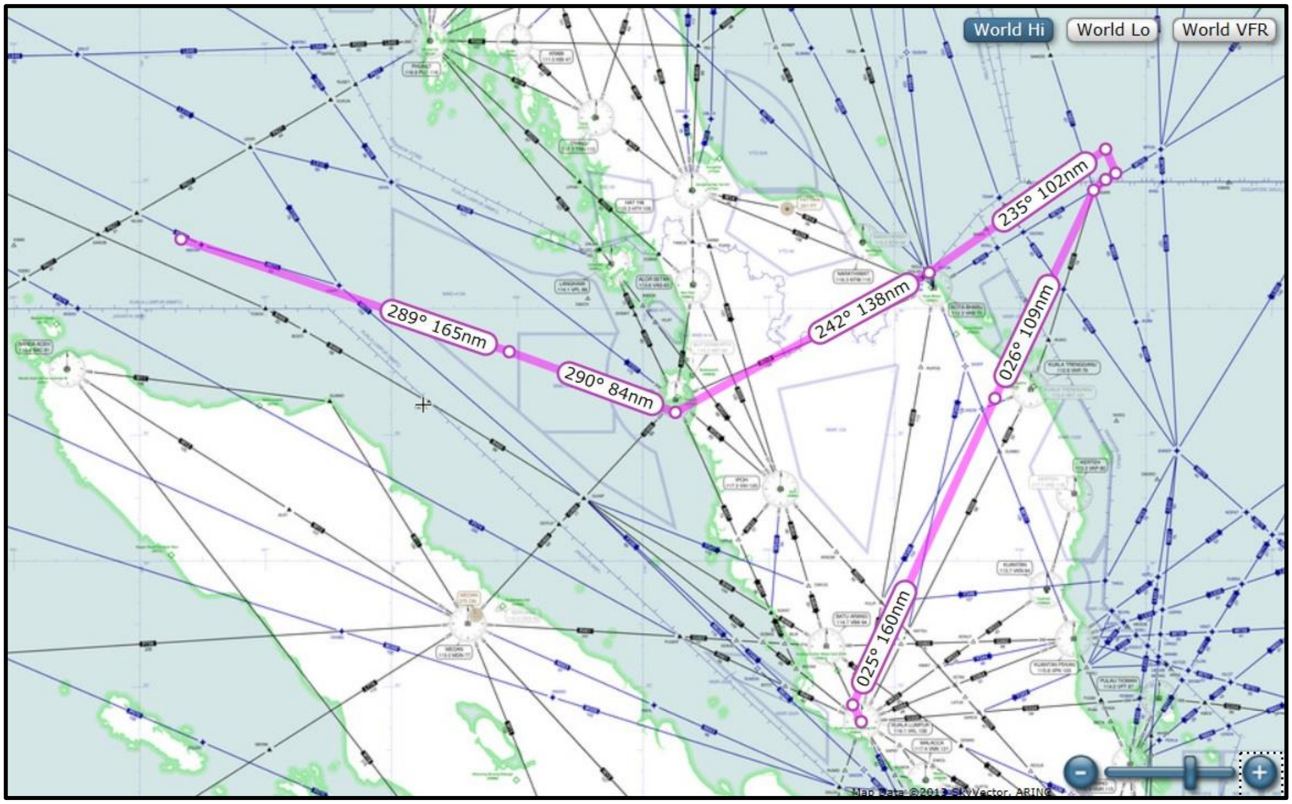
For precise post-diversion speed calculations I need to calculate the speeds corresponding to the complete radar track shown in the ATSB Report Figure 2.

5.3.2 Complete Radar Track Analysis

Table 5-1 shows the parameters for the portion of the MH370 route from takeoff at 16:41 UTC until radar contact is lost at 18:22:12 UTC. The aircraft positions are taken directly from Figure 2 in the ATSB Report except for Kuala Lumpur International Airport. Event times not known independently have been estimated to produce a steady true airspeed, but this does not materially affect the average true air speeds either before or after diversion. In Table 5-1 the True Air Speed boxes shaded yellow are before diversion, and the ones shaded green are after diversion.

A map of this radar track, based on the data in Table 5-1, is shown in Figure 5-3. It is effectively a digitized version of Figure 2 in the ATSB Report.

Figure 5-3 Map of Radar Track Based on ATSB Report Figure 2



The following paragraphs summarize the known parameters for each flight segment during that time period, and there is a row of entries in Table 5-1 corresponding to each flight segment.

5.3.2.1 KLIA

Takeoff from Kuala Lumpur International Airport (KLIA) occurred at ~16:41 UTC with a northwesterly bearing (~343 degrees). BTO and BFO readings prior to the takeoff time have been averaged in order to calibrate and to assess the accuracy of the BTO and BFO model predictions. I used geodetic coordinates (2.727N, 101.710E) for the BTO calibration check.

About 8 NM from KLIA at (2.86N, 101.67E), 9M-MRO turned right to a bearing of 25 degrees and headed to the first waypoint along its planned route – IGARI.

5.3.2.2 Last ACARS Data

9M-MRO reached (5.29N, 102.80E) at 17:06:43 UTC based on the last ACARS data.

5.3.2.3 Arrive IGARI

When 9M-MRO reached IGARI at ~17:20:39 UTC, it turned right and headed to its next planned waypoint – BITOD. IGARI is located at (6.937N, 103.585E).

5.3.2.4 Last Secondary Radar Data

The last secondary radar position was at ~17:21:37 UTC at (7.02N, 103.68E).

5.3.2.5 Begin Diversion Turn-Around

At approximately 17:22:20 UTC, 9M-MRO diverted from its planned route at (7.07N, 103.76E). A left turn of ~180 degrees was executed in order to head back to Malaysia.

5.3.2.6 End of Turn-Around

A tight U-turn was executed that ended at ~17:24:39 in a southwesterly heading at (7.26N, 103.68E). The distance traveled during this flight segment (19.3 NM as shown in Table 5-1) is the length of the circular turn, not the straight line range between the turn start and end positions.

5.3.2.7 Arrive Kota Bharu Airport

The nearest airport to 9M-MRO at the time of diversion is Sultan Ismail Petra Airport in Kota Bharu, Malaysia (VKB / WMKC). It has a 7,900 foot runway and commercial airline service. It appears that the MH370 flight crew initiated a path to one of the aviation waypoints immediately to the north of the airport. These waypoints form a tight cluster and include ABTOK, GOLUD, and KADAX. It is not necessary for my analysis to discern which waypoint was selected since the actual flight path is extremely close to all of them. I have chosen to use GOLUD at (6.285N, 102.278E), which is 8 NM north of Kota Bharu Airport. This location matches the radar track in the ATSB Report Figure 2.

The radar track from the turn-around to Kota Bharu is not perfectly straight. It meanders several NM on either side of a straight path. It is unclear whether this represents small errors in the measured radar positions (while the aircraft was being flown to Kota Bharu under Lateral Navigation control of the Flight Management System), or if the aircraft was being flown manually.

No landing attempt was made at Kota Bharu Airport.

5.3.2.8 Arrive Penang Airport

At or near Kota Bharu, 9M-MRO changes course slightly to the right and heads across Malaysia toward the Penang International Airport (VPG / WMKP) with its 11,000 foot paved runway. Again, the flight path is meandering a bit, possibly because it is being flown manually. 9M-MRO makes a slow turn to the right around the airport before settling on a straight northwesterly course into the Andaman Sea. The point of closest approach at 17:52:37 UTC to Penang International Airport is (5.18N, 100.26E) which is 6 NM to the south. Possibly the co-pilot was flying the aircraft manually from the right-hand seat, and he passed the airport so that it was visible through the cockpit windows on the right side. No landing attempt was made at Penang International Airport (now the second airport to be passed by).

5.3.2.9 Arrive Pulau Perak

Inspection of the radar track southwest of Penang indicates that the slow turn was continued to a point approximately 25 NM west of Penang Airport. Then the radar track becomes very straight indicating Lateral Navigation by computer. The end of the path passes extremely close to MEKAR, and it is possible that MEKAR was the selected navigation waypoint. Another possibility is ANOKO, since the extended track also passes very close to it, and ANOKO is a waypoint used for landing approach from the north to the airport at Banda Aceh, Indonesia. MEKAR is much closer than ANOKO and also puts the aircraft north of the tip of North Sumatra and the airport at Banda Aceh. For my air speed analysis, whether the selected waypoint was MEKAR OR ANOKO is immaterial, since I only use the ATSB Report Figure 2 radar track positions, not MEKAR or ANOKO coordinates.

From Figure 2 in the ATSB Report I get a Pulau Perak position of (5.66N, 98.94E) for 9M-MRO. The radar screen shot shown in China to the families indicates the time for passage by Pulau Perak is 18:02:33 UTC.

5.3.2.10 Last Radar Position

The last military radar position is (6.55N, 96.33E). The radar screen shot shown in China to the families indicates the corresponding time is 18:22:12 UTC.

This position is slightly past MEKAR, but the Flight Management System will continue the great circle course past the last waypoint entry if no subsequent entries were made by the flight crew.

5.3.3 Post-Diversion True Air Speed

Table 5-1 above summarizes the parameters for the flight segments in MH370's route from takeoff at 16:41 UTC until radar contact is lost at 18:22:12 UTC.

The purpose of analyzing these data is to answer four critical questions that bear on what 9M-MRO did after radar contact was lost:

a) What was the position of the last radar contact?
b) When was the last radar contact?
c) In what direction was 9M-MRO flying when it was last seen by radar?
d) At what true air speed was 9M-MRO flying when it was last seen by radar?

The first question is answered by the ATSB in Figure 2: the aircraft was at (6.55N, 96.33E).

The second question is the time of the last radar contact. The ATSB Report shows 18:22 UTC. Photographs of the radar screen shot shown to the families in China indicate a more precise time of 18:22:12 UTC, and I will use that time since the ATSB figure is probably just rounded off to the nearest minute.

The third question regarding bearing is answered by the angle of the final radar track in Figure 2 of the ATSB Report. It is very nearly 289 degrees, as shown in the last line of the upper section of Table 5-1.

The fourth and last question posed above (speed) is vital to my analysis. I believe the average post-radar true air speed should closely match the average true air speed during the (post-diversion) radar track. There is no reason to expect any speed change at this point in the flight except when accompanied by a significant descent for landing. A first estimate can be made by simply looking at the ground speed between the Pulau Perak radar position and the final radar position, and then compensating for wind. From Table 5-1 the estimated true air speed during this segment is 499 knots. The two end point locations are uncertain to ~2 NM each, and the times for each end are uncertain by ~5 seconds each. As shown in the next to last line in the lower section of Table 5-1, these uncertainties led to an estimated error of +/- 9 knots (1-sigma) in the true air speed estimate of 499 knots for that flight segment alone.

It is desirable to improve the accuracy of the estimated post-diversion average true air speed in order to get the most accurate answer possible to the fourth question posed above. Fortunately, more data are available from the ATSB Report Figure 2, and they are also analyzed in Table 5-1. Almost exactly one hour elapsed between diversion and last radar contact. This is three times longer than the single flight segment that I first used to estimate true air speed, and the precision of the second estimate will be much improved. As shown in the last line in the lower section of Table 5-1, the average true air speed between "Begin Diversion Turn-Around" and "Military Radar – Final Position" is 500 knots. Applying the same location and timing errors as before, we now have an average true air speed measured at 500 knots with an uncertainty of only +/- 3 knots (1-sigma). In Mach numbers, this is Mach 0.867 +/- 0.005. I conclude that the flight crew increased the aircraft true air speed from Mach 0.83 to Mach 0.87 immediately upon implementing the emergency diversion to the (first) nearest airport at Kota Bharu. Perhaps their initial strategy was to reach the nearest airport as quickly as possible, and implementing that strategy meant a fast U-turn and a high-speed return trip to Malaysia.

Matching the post-diversion true air speed of the radar track therefore means finding routes after the last radar position that have an average (and steady) true air speed in the range of 500 +/- 6 knots (2-sigma). Note also in Table 5-1 the steadiness of both the pre-diversion (highlighted in yellow) and the post-diversion (highlighted in green) true air speeds.

The fact that the flight crew did not immediately decrease the air speed nor descend to a lower altitude upon diversion argues against an initial decompression scenario. Possibly the cabin/cockpit filled with smoke/fumes from a fire onboard but the cabin pressure was being maintained at its normal level, and apparently the flight crew decided to maintain both altitude and speed until they were ready (and able) to attempt a landing.

5.3.4 Average True Air Speed of Fitted Routes

The next consideration for route selection is the average true air speed of the fitted (post-radar) routes, shown in Figure 5-4. The search areas and the longitudes of the route end points are plotted in the same fashion as in the previous figure. You will note that all Lateral Navigation curves show low speeds at the northeastern end of the 7th arc and much higher speeds at the southwestern end.

Figure 5-4 Average True Air Speed for All Routes

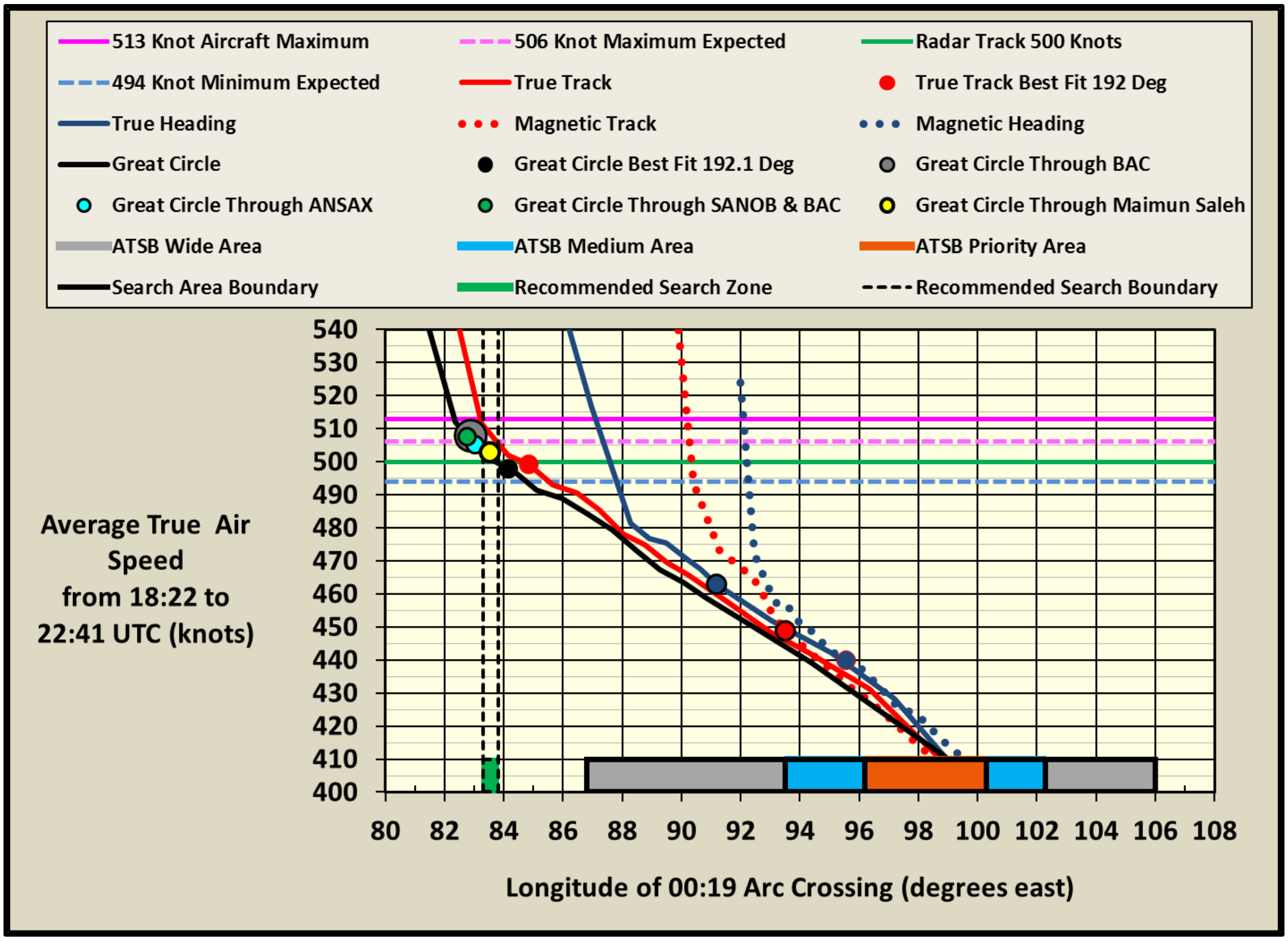


Figure 5-4 shows a horizontal green line at the expected speed of 500 knots (Mach 0.87) in order to match the radar track speed. There are pink and blue dashed lines at 506 and 494 knots, respectively, indicating 2-sigma boundaries for having matching speed based on uncertainties in the radar data. Satisfactory routes will therefore have average true air speeds typically between 494 and 506 knots.

Figure 5-4 demonstrates that several routes are within 2-sigma (+/- 6 knots) of the expected value of 500 knots. These include the true track at 192, the great circle at 192.1, and the waypoint /airport routes including the Maimun Saleh Airport great circle route at 192.3 degrees. The fact that there are any fitted routes, obtained by optimizing the steadiness (and not by fitting or forcing the average value) of the true air speed, that agree (within 1 %) with the 9M-MRO true air speed while being tracked by radar for an hour after diversion, makes these high-speed routes very credible indeed.

I would also point out that there is no route with very steady true air speed that both (a) matches the handshake arcs derived from the BTO data and (b) ends up either in the ATSB Priority Area or at the Independent Group's suggested location. All routes that end up east of 88 degrees longitude either have variable speed in order to hit the handshake arcs or miss the handshake arc positions badly in order to have constant speed. My proposed route through Maimun Saleh Airport, on the other hand, does both

simultaneously – it crosses the arcs precisely on time and has amazingly constant true air speed, plus it matches the post-diversion speed from the radar track data within 1 %.

5.4 <u>Measured and Predicted BFO's</u>

Table 5-2 summarizes selected Burst Frequency Offset (BFO) data for MH370 at times of critical flight events. The table shows the time, event, the aircraft parameters, the measured satellite BFO values, and my predicted BFO values for the Maimun Saleh Airport Route.

Table 5-2 Comparison of BFO Data for the Maimun Saleh Airport Route

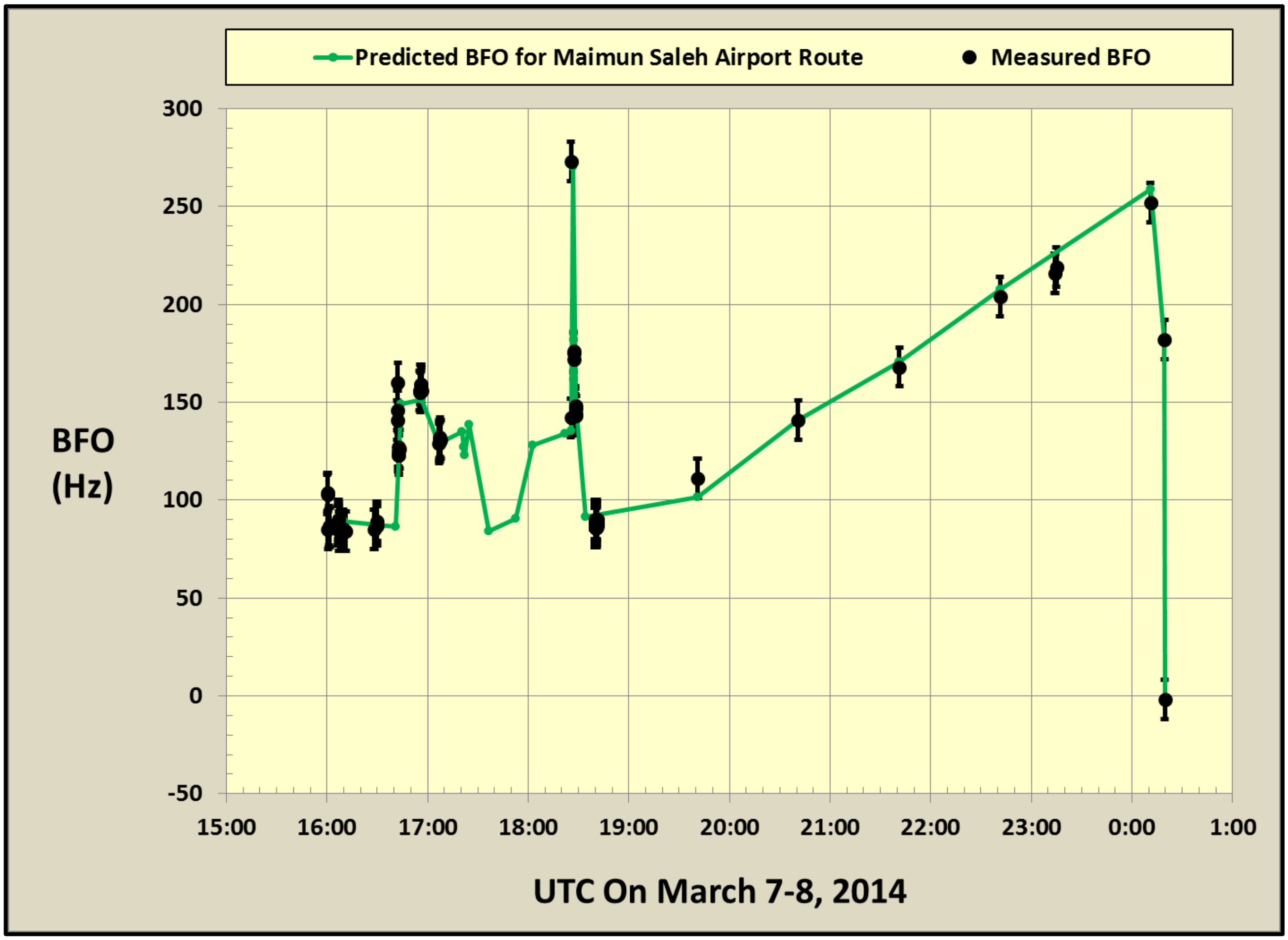
Time	Event	At Aircraft Position							BFO		
		Geocentric Latitude	Geodetic Latitude	Longitude	Altitude	Ground Speed	Bearing	Rate of Climb	Measured	Predicted	Error
UTC		degrees	degrees	degrees	feet	knots	degrees	feet / minute	Hz	Hz	Hz
2014/3/7 16:10:00	At Gate at KLIA Before Takeoff	2.709	2.727	101.710	69	0		0	86.8	89.3	2.4
2014/3/7 16:41:00	Takeoff from Kuala Lumpur Int. Airport (VKL)	2.709	2.727	101.710	69	0		0	87.0	86.3	-0.7
2014/3/7 16:44:00	Turn to IGARI	2.841	2.860	101.670	2,600	332	343.2	2,000		149.2	
2014/3/7 17:06:43	Last ACARS Data	5.255	5.290	102.800	35,000	469	25.0	0	131.5	129.1	-2.4
2014/3/7 17:20:39	Arrive IGARI	6.891	6.937	103.585	35,000	469	25.5	0		134.9	
2014/3/7 17:21:37	Last Secondary Radar Data	6.974	7.020	103.680	35,000	469	48.7	0		126.9	
2014/3/7 17:22:20	Begin Diversion Turn-Around	7.023	7.070	103.760	35,000	469	58.0	0		122.8	
2014/3/7 17:24:39	End Diversion Turn-Around	7.212	7.260	103.680	35,000	500	337.2	0		138.7	
2014/3/7 17:36:38	Arrive Kota Bharu Airport (GOLUD)	6.243	6.285	102.278	35,000	513	235.2	0		84.1	
2014/3/7 17:52:37	Arrive Penang International Airport (VPG)	5.146	5.180	100.260	35,000	512	241.3	0		90.7	
2014/3/7 18:02:33	Military Radar - Pulau Perak	5.622	5.660	98.940	35,000	506	289.9	0		127.9	
2014/3/7 18:22:12	Military Radar - Final Position	6.507	6.550	96.330	35,000	504	288.8	0		134.2	
2014/3/7 18:25:27	1st Handshake	6.653	6.698	95.897	35,000	505	288.8	0	142.0	135.5	-6.5
2014/3/7 18:27:00	Start of Final Turn	6.723	6.768	95.689	35,000	505	288.8	2,100		183.1	
2014/3/7 18:28:36	End of Final Turn	6.626	6.670	95.514	35,000	506	240.8	2,100		150.1	
2014/3/7 18:34:23	Maimun Saleh Airport (SBG/WITN)	5.835	5.874	95.340	35,000	508	192.4	0	87.8	91.6	3.8
2014/3/7 19:41:00	2nd Handshake	-3.375	-3.398	93.331	35,000	510	192.3	0	111.0	101.7	-9.3
2014/3/7 20:41:02	3rd Handshake	-11.662	-11.738	91.492	35,000	505	192.3	0	141.0	140.8	-0.2
2014/3/7 21:41:24	4th Handshake	-19.842	-19.965	89.563	35,000	501	192.5	0	168.0	170.6	2.6
2014/3/7 22:41:19	5th Handshake	-27.810	-27.969	87.476	35,000	493	193.1	0	204.0	207.6	3.6
2014/3/8 00:10:58	6th Handshake	-39.069	-39.258	83.893	15,000	449	193.9	0	252.0	258.6	6.6
2014/3/8 00:19:29	7th Handshake	-40.046	-40.236	83.530	1,000	400	195.9	-4,676	182.0	182.0	
2014/3/8 00:19:37	Extrapolated with 8 Seconds More Flight Time	-40.061	-40.251	83.524	100	400	195.9	-15,132	-2.0	-2.0	

My BFO predictions follow the methodology and nomenclature given in the ATSB Report (Appendix G). I used the ATSB Table 3 for the Inmarsat satellite location and velocity. I used an ellipsoidal model earth with WGS-84 parameter values and geocentric coordinates. I also used The ATSB Table 4 for the combined $\delta F_{sat} + \delta f_{AFC}$. I compared my BFO model calculations with ATSB Tables 5 and 6, and they match within a few tenths of one Hz. This confirmed the accuracy of my BFO model.

My predictions of BFO and its component terms are shown in greater detail in Table 9-8 in the Appendix.

The measured MH370 BFO data and my BFO predictions are plotted and compared in Figure 5-5. You will note the excellent agreement of my predictions with the BFO data for the entire route, including the two satellite phone calls at 18:39-18:40 and at 23:14-23:15 UTC in particular.

Figure 5-5 Measured and Predicted BFO Values



5.4.1 Residual BFO Errors

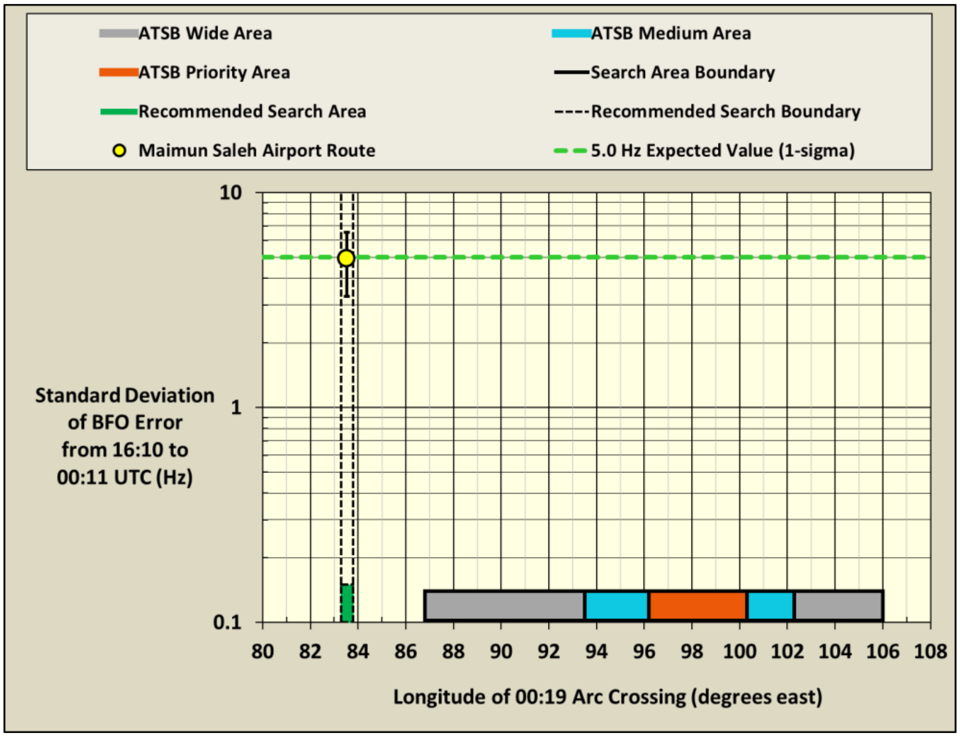
All individual MH370 BFO data measurements are plotted in Figure 5-5 as filled black circles with 10 Hz (2-sigma) error bars. My predicted BFO values for the Maimun Saleh Airport route are listed in Table 5-2 and plotted in Figure 5-5 as the green dots and line. You can see the excellent agreement of the BFO predictions and the measurements, both on the ground before takeoff and during the entire flight. The last two points at 00:19:29 and 00:19:37 UTC have been fitted manually by adjusting the Rate of Climb to be -4,700 and -15,100 feet per minute, respectively.

Excluding the last two BFO points from the calculation of error statistics, there are 10 remaining pairs of predicted and measured BFO values at matching times during the flight. The errors (= predicted BFO – measured BFO), listed in the last column in Table 5-2, have a mean of 0.0 Hz using a BFO Bias value of 149.5 Hz. My value for the BFO Bias agrees well with the value suggested by the ATSB of 150 +/- 5 Hz. The standard deviation of the 10 remaining BFO residual errors is 4.9 Hz with a 1-sigma uncertainty of 5/SQRT(10-1) = 1.6 Hz. This is in excellent agreement with the expected value of 5 Hz from the ATSB Report. Figure 5-6 displays the standard deviation of the residual BFO errors for the Maimun Saleh Airport route compared to the ATSB estimate. They are consistent, indicating that there are no significant systematic errors present in my BFO model.

I will reiterate that the Maimun Saleh Airport Route was determined using <u>only</u> the military radar data and the BTO data. Other than the manual fits to the last two BFO points(by solving for descent rates), the only free parameter in my BFO prediction is the BFO Bias, and my best-fit value for that parameter agrees

almost exactly with the value suggested by the ATSB. Therefore the excellent BFO match of my proposed route confirms its plausibility.

Figure 5-6 Standard Deviation of BFO Errors for Maimun Saleh Airport Route



It is clear from Figures 5-5 and 5-6 that the Maimun Saleh Airport Route is in excellent agreement with the MH370 BFO data, and that the bias and noise levels are each in excellent agreement with the values suggested in the ATSB Report. Therefore the Maimun Saleh Airport Route is an acceptable route in terms of being compatible with the MH370 BFO data.

5.5 **Final Turn Analysis**

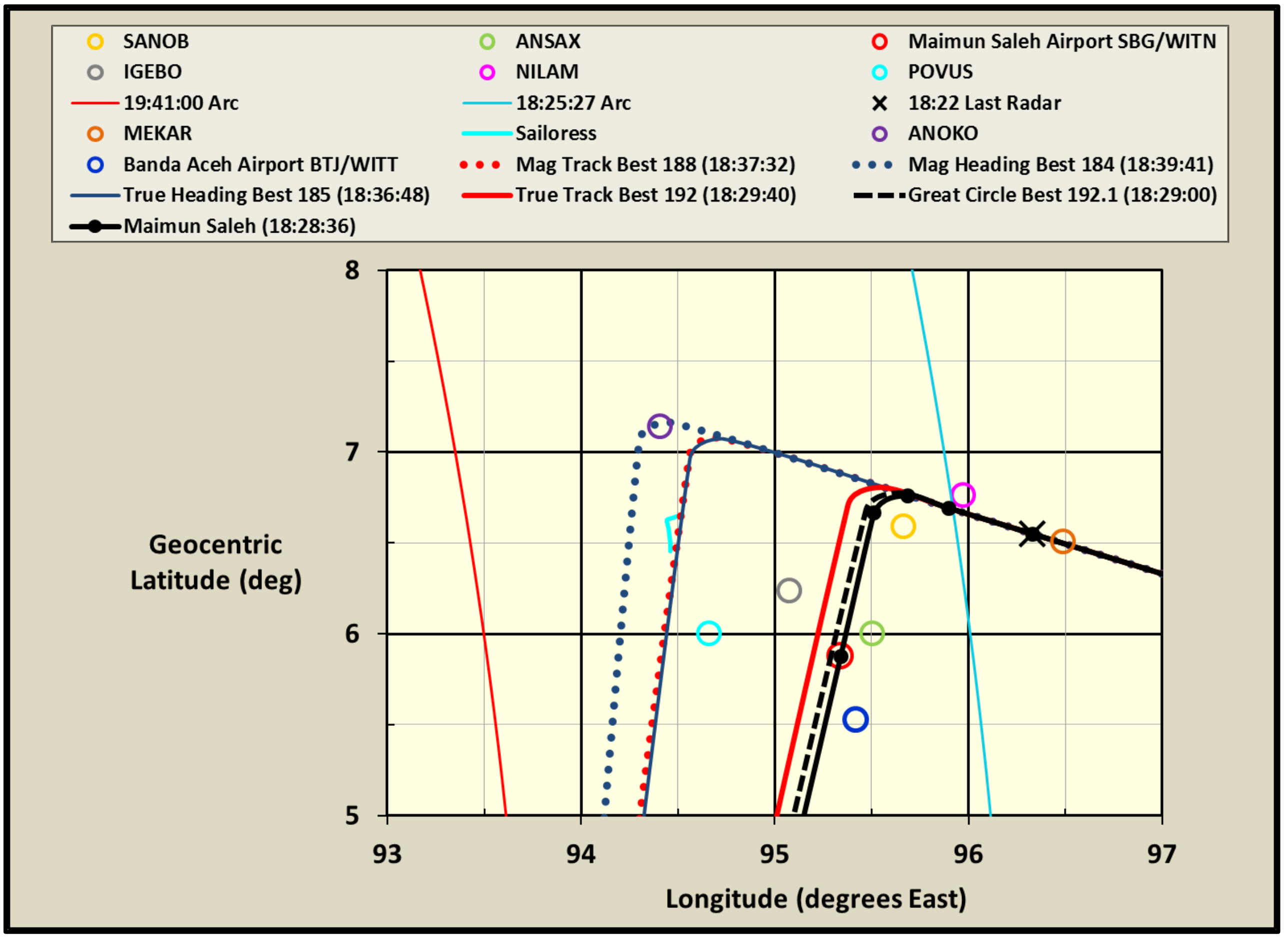
Figure 5.7 shows a map of the final turn area with an overlay of the best-fit routes for the various Lateral Navigation modes. 9M-MRO is moving from right to left in this diagram. The black "X" is the last military radar contact position. You will note that 9M-MRO passes through MEKAR as mentioned previously.

The legend in Figure 5-7 lists the names of the routes followed by their turn completion times in parentheses. Note that the most westerly route is the magnetic heading (blue dotted line), and its turn completion time is 18:39:41. That late turn time is clearly inconsistent with the BFO data near 18:28:10 UTC (which indicate a turn in progress). A detailed analysis of the BFO data near 18:28 UTC is presented in the next section.

The next two routes, moving to the right in Figure 5-7, are the magnetic track (red dotted line) at 18:37:32 UTC and the true heading (solid blue line) at 18:36:48 UTC. These routes are consistent with the 18:39 BFO data but not with the 18:28 BFO data.

Finally, we have the tight cluster of three routes for which the turns occur much earlier. The last to turn is the true track (red solid line) at 192 degrees (18:29:40). This route is not quite consistent with the 18:28 BFO data, but only by about one minute.

Figure 5-7 Map of Final Turn Area



We are left with the two remaining routes with the earliest turn times. The black dashed line is the best-fit 192.1 degree great circle route with a turn completion at 18:29:00. The solid black line is the Maimun Saleh Airport route with a turn completed at 18:28:36 UTC. Since the turn takes ~97 seconds to complete (from 289 degrees bearing to 192 degrees bearing at 1 degree / second), the turn completion times of both of these routes are consistent with a turn in progress at ~18:28:10 UTC and completed well before 18:39 UTC.

The location of the Sailoress (Kate Tee) is also shown in Figure 5-7. None of the satisfactory MH370 routes pass astern of her location (with an ENE heading) on that night. I therefore conclude the aircraft she saw was not 9M-MRO.

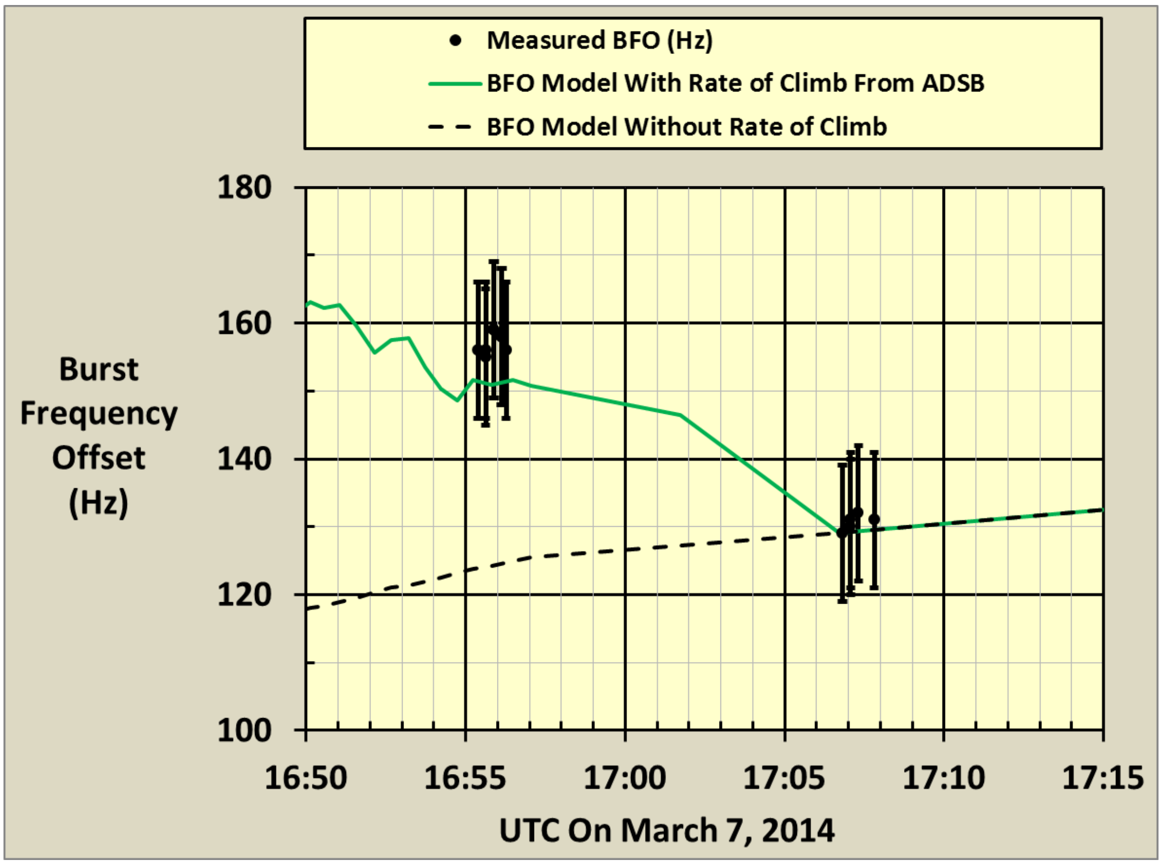
5.5.1 BFO Model of Final Turn

The MH370 BFO data imply a left turn near 18:27 UTC from a northwest bearing to a southern course (first turning toward the satellite and then away from it). That turn occurred at some point in time between 18:25 and 18:39 UTC, indicated by an initial rise and then a drop in BFO during that period of time. The initial rise is unexpected, because a left turn southward would cause only a drop in the BFO, stopping when the new southward bearing was reached. The rise and then fall in BFO might be due to a complex turn maneuver, but 9M-MRO would have to turn first to the right by 71 degrees to an approximate bearing of 0 degrees. Then it would have to make an almost immediate left turn of 168 degrees to get on its new 192 degree course to Weh Island. In my opinion this is not likely to have happened. I believe that the southward turn was instead accompanied by a climb in altitude.

First, in order to have confidence in my BFO predictions during ascents and descents, I need to validate the portion of the BFO model dealing with Rate of Climb. I have already demonstrated the validity of my BFO

model for different locations and bearings by consistency with the ATSB Report Appendix tables, so turns are accurately accounted for in my BFO model. Fortunately, we have the data for MH370 to validate the calculated effect of Rate of Climb on the BFO. During the period (shortly after takeoff) from 16:50 to 17:07 UTC, 9M-MRO was headed toward IGARI at a bearing of 25 degrees, and we have both ADSB data (including the Rate of Climb) and contemporaneous BFO measurements. I predicted the BFO using my model for this time period utilizing the aircraft location, altitude, speed, bearing, and Rate of Climb from the ADS-B data available from FlightAware. The results are shown as the green line in Figure 5-8.

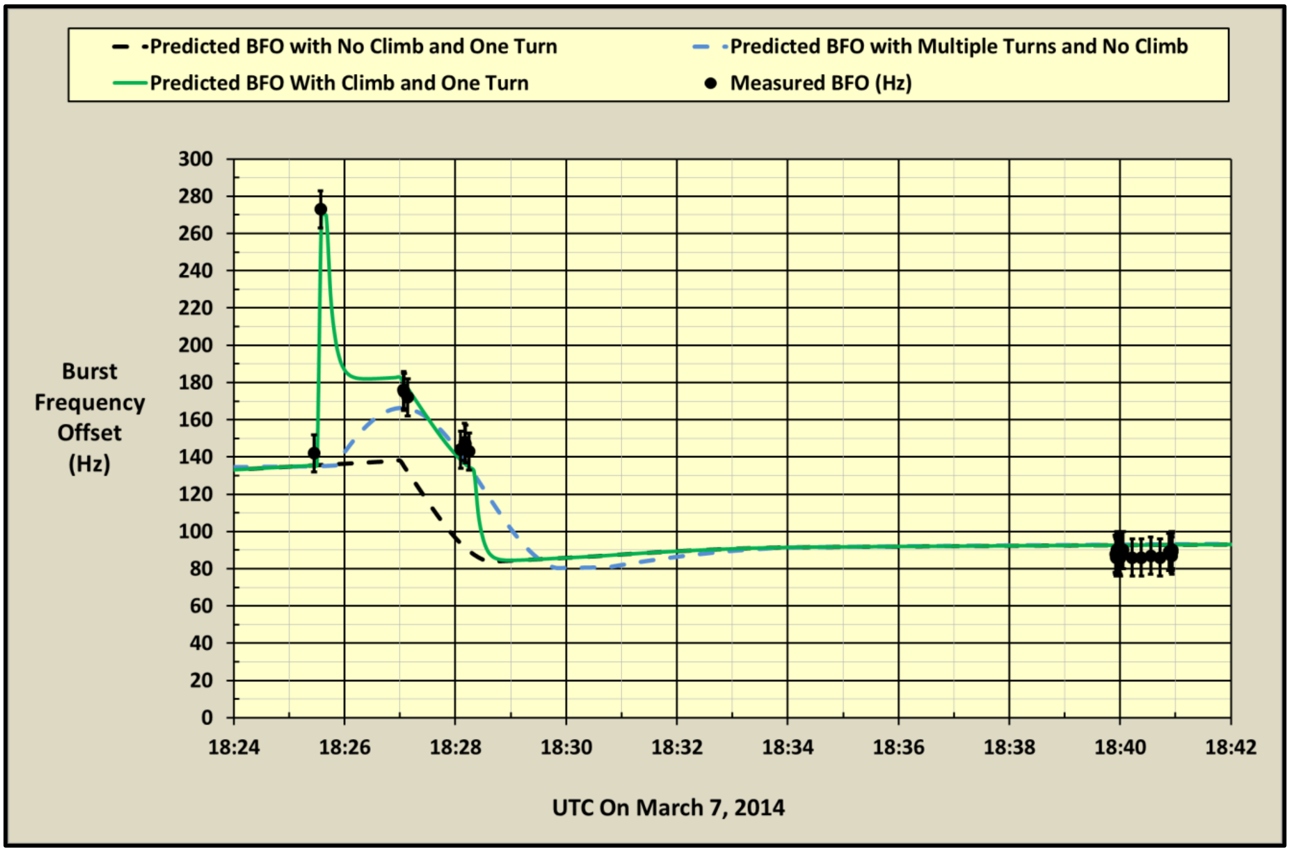
Figure 5-8 Validation of Rate of Climb BFO Model



In Figure 5-8 I have shown two BFO model predictions – one with the Rate of Climb included (the solid green line) and one without it (the black dashed line). At ~16:56 UTC the 9M-MRO Rate of Climb was ~1,300 feet per minute, and the effect on the BFO is an increase of ~27 Hz. You can see that the predicted BFO using the ADSB Rate of Climb matches the actual BFO recorded values, whereas without the additional term they do not match. This agreement therefore validates the Rate of Climb calculations in my BFO model. This portion of the BFO model will prove useful in understanding two critical events – the first at ~18:27 UTC when the southward turn occurred and the second at ~00:19 UTC during the final descent after fuel exhaustion.

A plot of the MH370 BFO data near the final turn time is shown in Figure 5-9, along with the usual 10 Hz 2-sigma error bars.

Figure 5-9 BFO Data at Final Turn



The question is "Can we determine precisely when the turn occurred using the BFO data between 18:25 and 18:40?"

The black dashed line is the predicted BFO assuming zero Rate of Climb. You can see that it predicts a smooth drop lasting the length of the turn (~1.5 minutes). There is no rise, and it does not match the middle three BFO data points. Thus a single turn alone is not consistent with the MH370 BFO data.

Allowing multiple turns can improve the fit to the MH370 BFO data. A turn sequence that improves the fit is shown as the blue dashed line in Figure 5-9. In this case, 9M-MRO is heading northwest at 289 degrees bearing. Then at ~18:25:50 UTC it turns to the right (at 1 degree per second) until it is moving due north (at 0 degrees). It stops turning to the right there and immediately turns back to the left until it is heading at 192 degrees toward Weh Island. The double-turn sequence takes 239 seconds to complete. The 142 Hz BFO datum at 18:25:27 UTC (the first handshake) does not indicate any turn or climb underway at that time. So the soonest the turn sequence could begin is immediately after 18:25:27 UTC, and the soonest it could be completed is 18:29:27 UTC. While the fit improved to the two points at 18:27 and 18:28 UTC, the fit is not improved to the 273 Hz BFO reading at 18:25:34 UTC.

The 273 Hz BFO at 18:25:34 is difficult to interpret. It corresponds to the second "Log-on/Log-off Acknowledge" message in a sequence that began just 7 seconds prior when the aircraft SDU logged on after power was restored to the SDU. The BFO at 18:25:27 of 142 Hz and the BTO at that time appear to be quite reasonable values. I can find no reason to conclude that the 273 Hz BFO measurement is unreliable since the value just 7 seconds prior appears to be reliable.

No turning of the aircraft can make the BFO go as high as 273 Hz at this point in the flight. The peak value of the blue dashed (double turn) line in the figure is at the highest possible BFO that any turn can make

(166 Hz). I therefore conclude that no turn sequence alone can produce a BFO consistent with the 273 Hz measurement.

It is possible for the aircraft to create rapidly changing BFO's by climbing or descending. We see this effect at 00:19 UTC, where the BFO drops precipitously in only a few seconds due to an accelerating descent. The reason for the high sensitivity of the BFO to vertical speed is simple. The Doppler frequency shift in the uplink path due to horizontal aircraft speed is almost completely removed by the Inmarsat airborne terminal in the aircraft, and only a very small percentage remains in the recorded BFO's. However, the airborne terminal does not compensate at all for vertical speed (=Rate of Climb). Therefore, the actual Rate of Climb of the aircraft can influence the BFO value by hundreds of Hz.

I have attempted to generate a simple scenario of altitude change that is consistent with the 273 Hz BFO as well as the lower values that occurred before and after. I would point out that Flight Level Changes, sometimes called step climbs, are routine on long-haul Boeing 777 flights. The Flight Management System may recommend a change to a higher altitude as the plane becomes lighter due to fuel consumption. This procedure improves fuel economy. Typical step altitudes for a Boeing 777-200ER are 31,000 / 35,000 / 39,000 feet, with 4,000 foot altitude steps. If this occurred on the MH370 flight, then perhaps the flight crew made a 4,000 foot altitude change at about the same time as power was restored to the SDU. The available satellite data are not sufficient to determine whether 9M-MRO was flying at 35,000 feet or at 39,000 feet altitude after the climb. We know for certain that 9M-MRO was at 35,000 feet when it began to divert. We also know that it did not descend to a much lower altitude because it maintained a very high air speed. It is possible that 9M-MRO descended slightly during its diversion and flyover of Malaysia, but this cannot be discerned by the primary radar tracking.

Typical climb rates after takeoff for a Boeing 777-200ER are in the range of 2,000 to 4,000 feet per minute. For the MH370 turn scenario, the BFO will increase by ~22 Hz for every 1,000 feet per minute of climb rate. A Rate of Climb of +6,000 feet per minute is needed to be fully consistent with the 273 Hz BFO. The aircraft may be capable of doing this for a brief period of time, but it is significantly higher than the typical (sustainable) climb rates.

Adding an altitude change before and during the turn resolves the initial appearance of BFO inconsistency.

The green solid line in Figure 5-9 is the predicted BFO assuming an increase in altitude took place beginning at 18:25:30 UTC, followed by a 97 degree left turn beginning at 18:27:00 UTC and ending at 18:28:36 UTC. The turn times are from the Maimun Saleh Airport route fit. The Rate of Climb needed to match all the data points is an initial (and very brief) ~6,000 feet per minute at 18:25:34 followed by a ~2 minute period of relatively constant ~2,000 feet per minute Rate of Climb. The altitude increase due to this climb scenario is roughly 4,000-5,000 feet. The green line in Figure 5-9 is the predicted BFO for a single left turn and the rates of climb scenario just described.

A more detailed comparison of the predicted BFO curve with the measured BFO data indicate that turn completion times between 18:28:26 and 18:29:06 UTC appear to be the limits of consistency with the BFO measurements. These limits are shown as red and blue dashed lines in Figure 5-10. The best estimate of the actual turn completion time, based on the BFO data, is 18:28:46 UTC with an uncertainty of +/- 20 seconds. The turn completion time for the Maimun Saleh Airport route fit is 18:28:36 UTC. It is shown by the green curve in Figure 5-10, and it falls between the earliest and latest turn times derived from the BFO data. I will repeat that the BFO data were not used in the route fitting process. The fact that the Maimun Saleh Airport route has a turn time that is within this +/- 20 second wide turn window derived from the BFO data is quite remarkable, and it provides great confidence that MH370 followed this particular route.

5.5.2 Final Turn Completion Times

Figure 5-11 shows the turn completion times for all the routes evaluated. In Figure 5-11 the horizontal green dotted line marks the 18:25:27 BFO data which indicates the turn has not yet started. The green and

pink dashed lines mark the earliest and the latest acceptable turn completion times allowed by the BFO data (from Figure 5-10). The SANOB/BAC/ANSAX routes are not acceptable because their turn times are too early. The true track at 192 degrees is not acceptable because its turn time is too late. The great circle at 192.1 degrees is marginally acceptable.

Figure 5-10 Turn Completion Time Limits

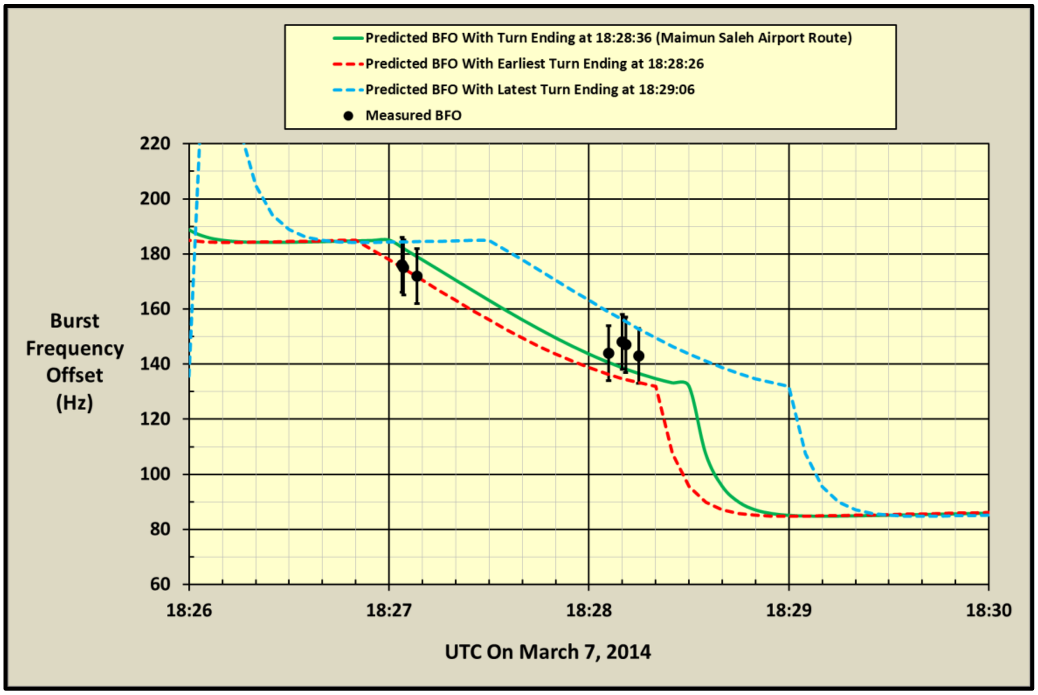
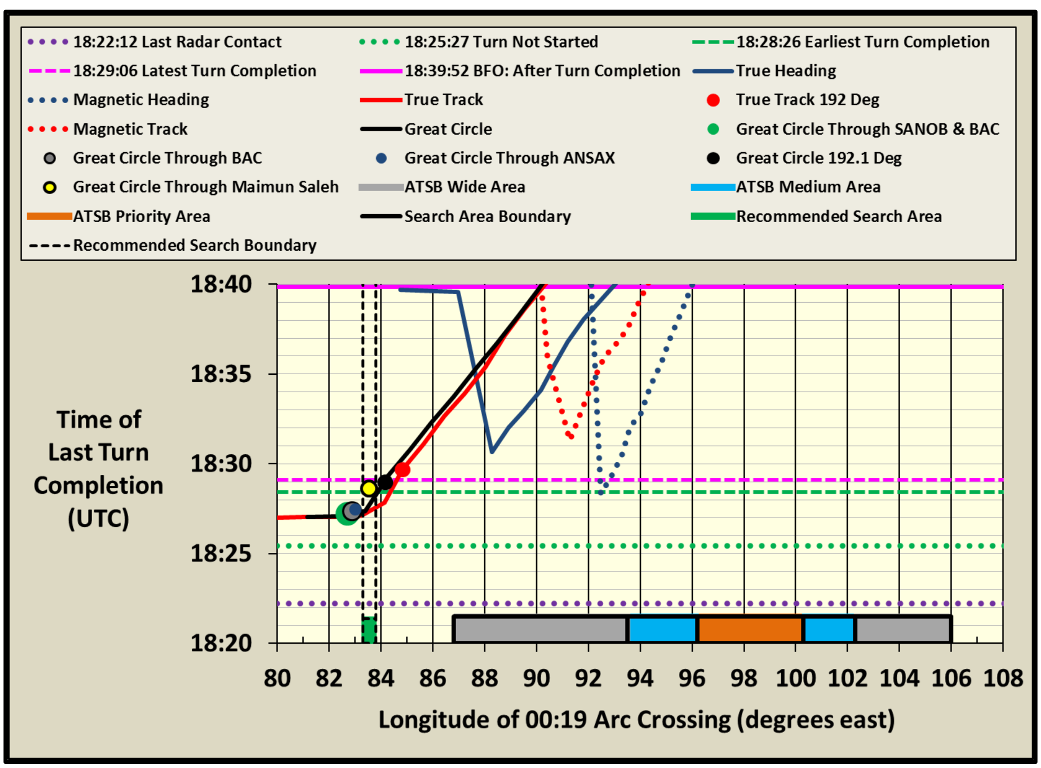


Figure 5-11 Times of Completion of Final Turn for All Routes



The Maimun Saleh Airport route is the best match to the acceptable turn times, falling in between the green and pink (dashed line) limits. This route has a turn completion time of 18:28:36 UTC, which is between the lower limit of 18:28:26 and the upper limit of 18:29:06 UTC derived from analyzing the BFO data. Since the great circle route at 192.1 degrees is essentially identical to the Maimun Saleh route, my conclusion is that the BFO turn data are only matched by the Maimun Saleh Airport route (and its "twin") and not by any of the other routes.

To summarize, I have shown the Maimun Saleh Airport Route has the following characteristics:

1) a great circle route with effectively zero bearing errors,
2) a RMS radial error less than 1.5 NM for all seven handshake arcs, easily consistent with the BTO noise level (26 μsec per the ATSB report),
3) a standard deviation of true air speed smaller than 1 knot (= extremely steady true air speed) from 18:22 to 22:41 UTC,
4) an average true air speed of 503 knots, consistent with the radar track data (500 +/- 6 knots 2 sigma),
5) a RMS BFO frequency error of 4.9 +/- 1.6 Hz, consistent with the expected BFO noise level (5 Hz 1 sigma per the ATSB Report), and
6) a final southward turn completion time of 18:28:36, consistent with the MH370 BFO data near that time within +/- 20 seconds.

6 Route Selection

6.1 Down-selection to the Identified Route

Table 6-1 lists ten parameters for the best-fit routes: the course in degrees, six constraint values, a viable route decision (does it meet all six constraints?), and the coordinates of the 00:19 UTC handshake arc intersection location. In the table, constraints that are satisfied are shaded green, and the ones that are not satisfied are shaded red. Inspection of the table indicates that there are three specific routes that meet five of the six selection criteria. These routes are nearly identical, and their end points form a tight cluster (as shown in Figure 5-1). Both the Maimun Saleh Airport route and its "twin" (the great circle 192.1 degree route) also satisfy the turn time constraint from the BFO data. As noted previously, a great circle route to an identified waypoint is much more likely to be correct than a route that is set solely by an initial bearing without an intended destination. Therefore the Maimun Saleh Airport route is identified as the unique MH370 route because it best satisfies all six constraints.

Table 6-1 Route Selection Criteria

Parameter			Constraint		Lateral Navigation Mode						Units
					Initial Bearing Set Using FMS		Set Using Mode Control Panel				
Name	Time Interval (UTC)	Metric	#	Description	Best-Fit Great Circle (GC)	Great Circle Through Maimun Saleh Airport	Best-Fit True Track (TT)	Best-Fit Magnetic Track (MT)	Best-Fit True Heading (TH)	Best-Fit Magnetic Heading (MH)	
Best-Fit Initial Bearing or Course	End of Final Turn to 00:19	Value	-	NA	192.12	192.36	192	188	185	184	degrees
		RMS Error	1	0.10 degrees or less ?	0.00	0.00	0.10	0.10	0.10	0.10	degrees
Handshake Arc Radial Errors	18:25 to 00:19	RMS Error	2	1.5 NM or less ?	1.50	1.50	1.50	1.50	1.14	1.50	nautical miles
Burst Frequency Offset	18:25 to 00:19	RMS Error	3	7.5 Hz or less ?	~5	4.9	~5	___	___	___	Hz
True Air Speed	18:22 to 22:41	Average	4	Within 6 knots (2-sigma) of 500 ?	498	503	499	449	463	439	knots
		Standard Deviation About Mean	5	3 knots or less ?	< 1	< 1	< 1	4	5	6	knots
Time of Final Turn Completion	NA	Value	6	Between 18:28:26 and 18:29:06 UTC ?	18:29:00	18:28:36	18:29:40	18:37:32	18:36:48	18:39:41	UTC
Is this a viable route ?				Are all 6 constraints satisfied ?	Yes	Yes	No	No	No	No	
00:19 UTC Handshake Arc (#7) Coordinates				Geocentric Latitude	39.80	40.05	39.51	34.27	35.98	32.15	degrees south
				Longitude	84.14	83.53	84.83	93.51	91.19	95.93	degrees east

Table 6-2 Maimun Saleh Airport Route

Time	Location or Event	Aircraft Position				Leg From Previous Position					At Aircraft Position		Calculations Using BTO Data From Inmarsat Data Communications Log		
									Using Average of Winds at Leg Ends		Wind				
		Geodetic Latitude	Geocentric Latitude	Longitude	Altitude	Range	Initial Bearing	Average Ground Speed	Average True Air Speed (TAS)	Average True Heading	Direction	Speed	Burst Timing Offset (BTO)	Handshake Arc Radius Calculated from BTO Using ATSB Equation, Ellipsoidal Earth, and Average Latitude-Dependent Earth Radius	Radial Error = Great Circle Distance of Aircraft from Sub-Satellite Point Minus Handshake Arc Radius
(UTC)		(deg N)	(deg N)	(deg E)	(ft)	(nm)	(deg)	(kts)	(kts)	(deg)	(deg)	(kts)	(µsec)	(nm)	(nm)
2014/3/7 16:41:00	Takeoff from Kuala Lumpur Int. Airport (VKL)	2.727	2.709	101.710	69						69	18.2	14,840	2,235.1	-0.68
2014/3/7 16:44:00	Turn to IGARI	2.860	2.841	101.670	2,600	8.30	343.17	165.9	168.3	349.3	69	18.2			
2014/3/7 17:06:43	Last ACARS Data	5.290	5.255	102.800	35,000	160.13	25.03	422.9	435.1	26.5	67	14.6	15,600	2,307.4	-0.16
2014/3/7 17:20:39	Arrive IGARI	6.937	6.891	103.585	35,000	108.95	25.51	469.1	478.0	26.4	68	8.8			
2014/3/7 17:21:37	Last Secondary Radar Data	7.020	6.974	103.680	35,000	7.54	48.73	469.7	478.0	49.1	68	8.8			
2014/3/7 17:22:20	Begin Diversion Turn-Around	7.070	7.023	103.760	35,000	5.63	57.97	469.3	478.0	58.2	68	8.8			
2014/3/7 17:24:39	End Diversion Turn-Around	7.260	7.212	103.680	35,000	19.33	337.19	499.9	499.9	338.2	68	8.8			
2014/3/7 17:36:38	Arrive Kota Bharu Airport (GOLUD)	6.285	6.243	102.278	35,000	101.97	235.18	510.6	500.1	235.1	54	12.5			
2014/3/7 17:52:37	Arrive Penang International Airport (VPG)	5.180	5.146	100.260	35,000	137.53	241.33	516.1	500.1	241.6	51	19.9			
2014/3/7 18:02:33	Military Radar - Pulau Perak	5.660	5.622	98.940	35,000	84.03	289.94	507.9	500.0	291.6	45	14.0			
2014/3/7 18:22:12	Military Radar - Final Position	6.550	6.507	96.330	35,000	164.80	288.81	503.2	499.4	289.7	50	3.0			
2014/3/7 18:25:27	1st Handshake	6.698	6.653	95.897	35,000	27.33	288.81	504.5	503.0	289.1	50	3.0	12,520	1,906.5	-1.76
2014/3/7 18:27:00	Start of Final Turn	6.768	6.723	95.689	35,000	13.08	288.81	504.5	503.0	289.1	50	3.0			
2014/3/7 18:28:36	End of Final Turn	6.670	6.626	95.514	35,000	13.49	240.82	505.9	503.0	240.9	50	3.0			
2014/3/7 18:34:23	Maimun Saleh Airport (SBG/WITN)	5.874	5.835	95.340	35,000	48.67	192.36	506.0	503.0	191.7	81	10.2			
2014/3/7 19:41:00	2nd Handshake	-3.398	-3.375	93.331	35,000	566.57	192.34	510.2	503.0	190.6	74	23.9	11,500	1,757.4	-0.11
2014/3/7 20:41:02	3rd Handshake	-11.738	-11.662	91.492	35,000	509.95	192.30	509.7	503.0	190.4	90	12.2	11,740	1,795.8	1.66
2014/3/7 21:41:24	4th Handshake	-19.965	-19.842	89.563	35,000	504.01	192.54	501.0	503.0	192.3	244	10.8	12,780	1,950.0	2.45
2014/3/7 22:41:19	5th Handshake	-27.969	-27.810	87.476	35,000	492.19	193.06	492.9	503.0	196.2	269	47.8	14,540	2,189.1	-1.97
2014/3/8 0:10:58	6th Handshake	-39.258	-39.069	83.893	15,000	699.34	193.91	468.1	489.0	198.3	242	39.3	18,040	2,611.1	0.00
2014/3/8 0:19:29	7th Handshake	-40.236	-40.046	83.530	1,000	61.00	195.89	429.8	458.1	199.4	242	39.3	18,400	2,650.8	0.00

The significant parameters associated with the Maimun Saleh Airport route are shown in Table 6-2. The table begins with the takeoff from Kuala Lumpur international Airport at 16:41 UTC on March 7, 2014 and goes until the last satellite data are received at 00:19 UTC on March 8, 2014. In this table, the calculated true air speeds are highlighted in green. Note the constancy. The radial arc crossing errors are highlighted in pink. Note the small values. The end point coordinates are highlighted in blue.

6.1.1 Geocentric and Geodetic Latitudes

In Table 6-2 I have listed both the geodetic and the geocentric coordinates of the Maimun Saleh Airport route. My EXCEL model fits routes using geocentric latitudes, as shown in Table 9-6 in the Appendix. I have converted the geocentric coordinates to geodetic coordinates to allow direct comparison with maps and ATSB proposed search areas. The geodetic coordinates of the 00:19 handshake arc crossing are (40.24S, 83.53E).

6.1.2 Maimun Saleh Airport Route Characteristics

The statistical properties of the Maimun Saleh Airport route are shown in Table 6-3. The ground and air speeds are calculated using 8 flight segments crossing a total of 5 handshake arcs. The last two arcs are not used in this case because the aircraft speed decreased while moving toward arcs #6 and #7, probably due to fuel exhaustion, first in one engine and then in both engines. You can see in Table 6.2 above how the ground and air speeds are dropping in the last two lines (arcs #6 and #7).

Note in Table 6-3 that the ground speed has a standard deviation of almost 7 knots, but the true air speed varies by less than 0.1 knots. This large reduction factor is a characteristic of all nearby routes, not just this route. That fact confirms that the wind data are accurate and the wind model is effective, and this allows much greater discrimination of the routes with the steadiest true air speed than would otherwise be possible. Note also in Table 6-3 that the seven arc crossings have a worst-case radial error of only 2.6 NM and a standard deviation of 1.5 NM (equivalent to ~13 μsec RMS BTO error).

Table 6-3 Statistics for Maimun Saleh Airport Route

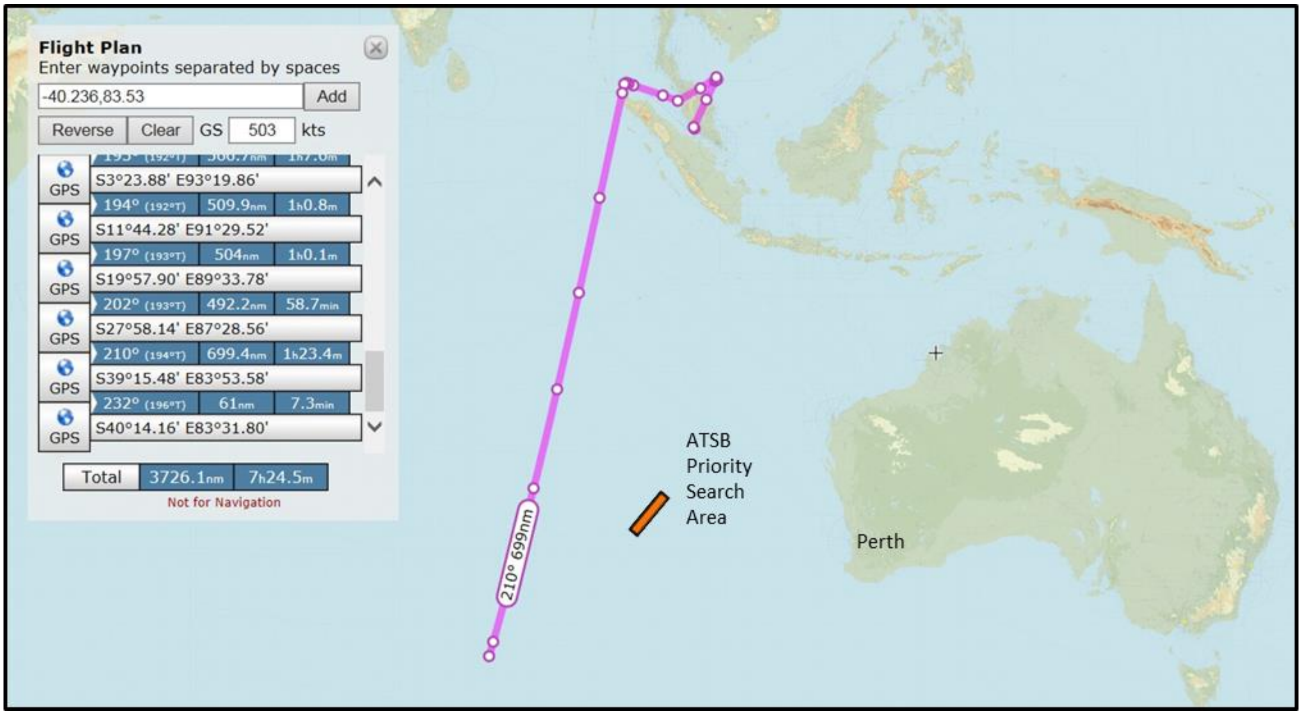
Great Circle Route Through Maimun Saleh Airport								
From / To		Parameter Name	Best-Fit Parameter Statistics					
# Handshake Arc Crossings	UTC		Average (Mean)	Minimum	Maximum	Standard Deviation	Units
5	18:22 - 22:41 UTC	Ground Speed	503.7	492.9	510.2	6.9	knots
5	18:22 - 22:41 UTC	True Air Speed (TAS)	503.0	503.0	503.0	0.0	knots
7	18:25 - 00:19 UTC	Radial Errors at Arc Crossings	0.04	-1.97	2.45	1.50	NM

6.1.3 Maximum Aircraft Range

The entire flight route for MH370 is shown in Figure 6-1. Note the end point's location relative to Australia and to the current ATSB Priority Search Area.

The total distance traveled during the MH370 flight is approximately 3,735 NM over the ground and 3,765 NM through the air. The Maimun Saleh Airport route end point is approximately 60 NM beyond the (extrapolated) ATSB "Performance Limit" shown in Figure 20 in the ATSB Report. The ATSB did not specify in detail how their Performance Limit estimate was calculated. I would expect the errors in this type of calculation to be on the order of several percent. Since the total range difference of 60 NM is only 1.6% of the total distance traveled, I conclude that this route is most likely within the aircraft range capability taking into account the expected accuracy of the maximum range calculations

Figure 6-1 Complete MH370 Flight Route



7 Search Zone Definition

7.1 Search Zone Longitude Range

As seen in Table 6-1 above, the Maimun Saleh Airport route satisfies all the constraints. The next question is "How accurate is the prediction of the 7th arc crossing location?"

I have investigated the sensitivity of the fitted end point to small shifts (several nautical miles) in the starting point at 18:22 UTC. I found the end point longitude varied by less than +/- 0.1 degrees (about +/- 5 NM). In addition to these small random errors, I would expect there are small systematic computational errors of comparable magnitude, as well as some potential unknown error sources. Allowing for these known and unknown errors, I would say that there is a high probability of finding 9M-MRO within +/- 15 NM along the 7th arc from 83.53 degrees east longitude. This corresponds to the range of 83.20 – 83.80 degrees east longitude.

The recommended search zone for 9M-MRO extends 32 NM from 83.2 to 83.8 degrees east longitude along the 7th handshake arc.

7.2 Search Zone Width

The recommended search zone is approximately 32 NM in length. In determining the optimum "width" of the Priority Search Area, I would note that the BFO data indicate a very rapid descent rate at 00:19 UTC. Because a total loss of engine power occurred several minutes earlier near 00:15:49 UTC, it is likely that the aircraft had already descended to a much lower altitude prior to 00:19 UTC.

7.2.1 Final Descent

It appears that the fuel was exhausted in one of the engines before 00:11 UTC, and this caused the slowdown seen in the calculated average speed between 22:41 and 00:11 UTC. A few minutes later, at approximately 00:15 UTC, the fuel was exhausted in the second engine (causing a temporary loss of electrical power), and 9M-MRO began to slow down and descend more quickly. The BFO data at 00:11 UTC indicate no large speed or bearing changes compared to 22:41 UTC. The BFO data at 00:19:29 UTC, however, indicate either a descent rate of up to ~4,700 feet per minute (at most), or perhaps a slightly smaller descent rate coupled with a bearing change to a direction directly away from the satellite. By 00:19:37 UTC, only 8 seconds later, the BFO data indicate an apparent descent of ~15,100 feet per minute (or less if the bearing changed also). During this brief period the aircraft has accelerated toward the ocean surface at most by ~0.7G's. Because the average ground speed between 00:11 and 00:19 UTC was still over 400 knots, I believe that 9M-MRO was still gliding from 00:15 until nearly 00:19 UTC in the same general direction as it was going during powered flight. In any case, expected transmissions after 00:19:37 did not occur, presumably because of impact a few seconds later and within a mile or two of the handshake arc.

Estimation of the altitude of 9M-MRO at the 7th arc is useful because the calculated arc position has a small contribution from the aircraft altitude. The BTO and BFO data are insufficient to precisely determine the aircraft altitude or heading at the 7th arc. I would make a rough estimate of 1,000- 1,500 feet altitude based on (a) the fact that the aircraft was descending already, (b) it lost as much as ~700 feet on top of that descent rate in the 8 seconds between the two 00:19 BFO's, and (c) SDU transmissions ceased very shortly thereafter.

7.2.2 Location Error Analysis

The location of the 7th arc is uncertain by approximately +/- 3 NM because of the noise in its BTO value. To insure a high probability of success, a reasonable estimate of the half-width of the new search zone is 3 NM (from BTO noise) + 1 NM (8 seconds travel distance) + 3 NM (systematic computational errors) + 3 NM (unknown errors) = 10 NM. The width of the recommended search zone with respect to the 7th arc is therefore +/- 10 NM = 20 NM, with a slightly higher probability of success expected in the half outside the arc.

7.3 <u>Search Zone Area</u>

The recommended search zone is 32 NM long X 20 NM wide = 640 square NM or ~2,200 km^2. This is only 3.7% of the ATSB's currently planned search area of 60,000 km^2. I believe the 9M-MRO wreckage is very near my calculated impact point. The amazing precision of the satellite data coupled with my new analysis techniques will allow a targeted investigation to be carried out within a relatively short period of time and using only a fraction of the available assets and funds. Other factors enabling an efficient and rapid search in this new area are the reduced ocean depth and the flatter seabed compared to the Broken Ridge region now being mapped.

8 Future Work

The ATSB has attempted, with limited success, to replicate the routes of other B777 flights out of Kuala Lumpur. It would be instructive to apply my method to data from those flights in order to assess the accuracy of predicted locations. A direct comparison of predicted and known aircraft locations would then be possible, and this comparison would shed light on the systematic and random errors affecting the accuracy of my prediction. If satellite and "ground truth" data were made available for a similarly simple but lengthy route, it may be possible to validate the accuracy of my method.

9 Appendix

9.1 <u>Mode Control Panel Routes</u>

Tables 9-1 to 9-4 are EXCEL worksheets showing the parameters for the best-fit routes for each of the four Lateral Navigation modes operable through the Mode Control Panel. The tables contain (at each handshake arc crossing or other event) the time, aircraft position, ground speed, wind, air speed, heading, magnetic declination, handshake arc radius, and radial error.

Table 9-1 Best-Fit True Track Route at 192 Degrees

MH370 Satellite Data Fitting Results
Dr. Bobby Ulich - 2014.09.04

Constant True Track Solution (Track = 192.0 deg)

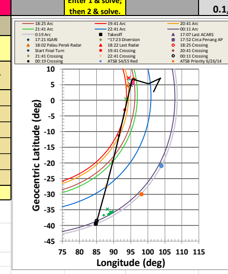
Time Stamp (UTC)	Notes	Aircraft Position: Geocentric Latitude (deg N)	Aircraft Position: Longitude (deg E)	Aircraft Position: Altitude (ft)	Leg From Previous Position (Assume Rhumbline): Range (nm)	Bearing (deg)	Average Ground Speed (kts)	Using Average of Winds at Leg Ends: Average True Air Speed (TAS) (kts)	Using Average of Winds at Leg Ends: Average True Heading (deg)	Wind at Aircraft Position: Direction (deg)	Wind at Aircraft Position: Speed (kts)	Magnetic Declination at Aircraft Position (West of North) (deg)	Magnetic Declination at Midpoint Between This Aircraft Position and Previous Position (West of North) (deg)	Average Magnetic Heading During Segment By Averaging Start, Midpoint, and End Declinations (deg)	BTO from Inmarsat Log (microsec)	Handshake Arc Radius Calculated from BTO Using ATSB Equation, Ellipsoidal Earth, and Latitude-Dependent Average Earth Radius (nm)	Radial Error = Great Circle Distance of Aircraft from Sub-Satellite Point Minus Handshake Arc Radius (nm)	At Aircraft Position: rac = Ellipsoidal Earth Radius At This Aircraft Latitude (km)	At Aircraft Position: Average Earth Radius Integrated From Approximate Sub-Satellite Point to This Aircraft Latitude (Used in Handshake Arc Radius Calculation) (km)	Satellite Position From ATSB Report: X (km)	Satellite Position From ATSB Report: Y (km)	Satellite Position From ATSB Report: Z (km)
2014/3/7 16:41:00	KLIA (VKL)	2.727	101.710	69						69	18	0.1			14840	2235.1	-0.64	6378.1	6378.2	18120.3	38081.2	748.7
2014/3/7 17:07:00	Last ACARS Data	5.29	102.80	35,000	167.3	23.0	386	398	24.65	67	15	0.2	0.2		15600	2307.4	0.01	6378.0	6378.3	18125.3	38079.7	958.0
2014/3/7 17:21:00	Arrive IGARI	6.94	103.59	35,000	109.5	25.4	469	478	26.29	68	9	0.4	0.3					6377.8				
2014/3/7 17:23:00	Estimated Diversion Position (en route BITOD)	7.07	103.81	35,000	15.9	58.8	477	486	58.98	68	9	0.4	0.4					6377.8				
2014/3/7 17:51:45	Circa Penang AP (VPG)	5.26	100.27	35,000	238.1	242.8	497	483	242.94	51	20	0.4	0.4					6378.0				
2014/3/7 18:02:00	Malaysian Military Radar - Pulau Perak	5.70	98.92	35,000	84.9	288.2	497	488	289.86	45	14	0.5	0.4					6377.9				
2014/3/7 18:22:12	Malaysian Military Radar - Final Position	6.55	96.33	35,000	163.1	288.3	485	481	289.15	50	3	0.7	0.6					6377.9				
2014/3/7 18:25:27	1st Handshake	6.69	95.90	35,000	27.1	288.3	501	499.2	288.55	50	3	0.8	0.7		12520	1906.5	-1.34	6377.8	6378.3	18136.7	38071.8	1148.5
2014/3/7 18:28:04	Start of Final Turn	6.81	95.55	35,000	21.9	288.3	501	499.2	288.55	50	3	0.8	0.8					6377.8				
2014/3/7 18:29:40	End of Final Turn	6.71	95.38	35,000	13.4	240.3	501	499.2	240.27	135	0	0.8	0.8					6377.8				
2014/3/7 19:41:00	2nd Handshake	-3.06	93.33	35,000	599.9	191.8	505	499.2	190.479	74	24	1.4	0.8	191.48	11500	1757.4	-2.99	6378.1	6378.0	18145.1	38067.0	1206.3
2014/3/7 20:41:02	3rd Handshake	-11.34	91.57	35,000	508.7	191.9	508	499.2	190.263	58	11	3.6	2.2	192.68	11740	1795.8	-2.17	6377.3	6377.1	18152.1	38064.0	1159.7
2014/3/7 21:41:24	4th Handshake	-19.53	89.77	35,000	502.8	192.0	500	499.2	192.005	244	11	8.2	5.6	197.83	12780	1950.0	-0.01	6375.8	6375.9	18159.5	38061.3	1033.8
2014/3/7 22:41:19	5th Handshake	-27.50	87.91	35,000	489.7	192.0	490	499.2	195.391	272	51	15.8	11.6	207.27	14540	2189.1	-0.50	6373.6	6374.3	18167.2	38058.3	837.2
2014/3/8 0:10:58	6th Handshake	-38.53	85.09	15,000	677.5	192.1	453	474	197.033	240	43	33.0	23.4	221.09	18040	2611.2	0.00	6369.9	6371.8	18177.5	38051.7	440.0
2014/3/8 0:19:29	7th Handshake	-39.51	84.83	5,000	59.6	192.0	420	450	196.087	240	43	35.0	34.0	230.09	18400	2651.5	0.00	6369.5	6371.5	18178.4	38050.8	390.5

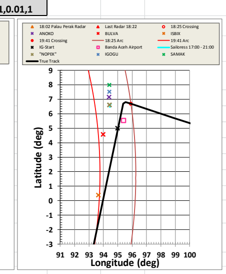
	Average Ground Speed	501.0	AVG TAS = Last Radar to 22:41	499.2	KTS	AVG True Track Weighted by Range From End Turn to 00:19	191.96
RUN # TTXXX	St Dev Ground Speed	6.5	STDEV TAS = Last Radar to 22:41	0.00	KTS	STDEV True Track About Desired G34 From End Turn to 00:19	0.100
			AVG TAS End Turn to 20:41	499.2	KTS	RMS Radial Arc Error (nm) = All 7 Arcs	1.50
AVG TAS 499.2 knots			AVG TAS 20:41 to 22:41	499.2	KTS	MAX Radial Arc Error (nm) = All 7 Arcs	2.99
STDEV TAS 0.00 knots							

Course	AVG TAS	StDev TAS	Turn Time	Lat	Lon	Max TAS	AVG GS	StDev GS
192.00	499.2	0.00	18:29:40	-39.51	84.83	499.2	501.0	6.5

	Mode=1 Gain	Objective Function To Be Minimized Using Solver	Weighting Coefficients	ST DEV	Objective Terms	MAX	Flag	StDev / Max Limit	Mode=2 Gain	Mode =1 Limit Scalar	Mode=1 Lowest Limit	Mode=2 Limit
100,7,0.1	100	True Track Error	0.001	0.100	0.000	0.199	0	0.10	0.0010	1.40	1.00	0.10
	7.0	Radial Error	0.0030	1.50	0.004	2.988	1	1.50	0.0030	1.40	1.10	1.50
	0.1000	True Air Speed	1.000	0.0002	0.000	499	2	1500	1.00			
	Objective Cell to Minimize in Solver			0.005			Enter 1 & solve; then 2 & solve.		0.1,0.01,1			

			Rad Err	TAS	Fitted Parameters								
RMS Radial Error	1.50	nm											
MAX Radial Error	2.99	nm	-1.336	499	27.13	Last Radar to 18:25 Arc	(nm)						
				499	21.90	18:25 to Start of Turn	(nm)						
AVG True Track	191.96	deg		18:28:04	18.4679	Turn Start Time	(Hours)	>	18.4250	and	<	19.6167	hours
STDEV True Track	0.10	deg		499	288.25	True Bearing Into 18:25 &Turn	(deg)		18:25:30	=Soonest Allowable Turn Start		19:37:00	=Latest Allowable Turn Start
MAX True Track Error	0.20	deg			192.00	True Track Out of Turn To 00:19	(deg)	Fitted Bearing To:		SAVED PARMS	Desired True Track	Actual True Track	True Track Error
			-2.99	499	599.85	End Turn to 19:41	(nm)	191.80	19:41	180.78	192.00	191.80	0.20
			-2.17	499	508.74	19:41 To 20:41	(nm)	191.93	20:41	181.74	192.00	191.93	0.07
			-0.01	499	502.86	20:41 To 21:41	(nm)	191.97	21:41	179.34	192.00	191.97	0.03
			-0.50	499	489.82	21:41 To 22:41	(nm)	192.02	22:41	175.29	192.00	192.02	-0.02
			0.00	474	677.66	22:41 To 0:11	(nm)	192.06	0:11	165.07	192.00	192.06	-0.06
			0.00	450	59.61	0:11 To 0:19	(nm)	191.98	0:19	157.39	192.00	191.98	0.02
					506.02	Turn Ground Speed	(kts)					MAX TT ERR	0.20





Range and Bearing Calculations for Each Leg (Assume Rhumbline)

Assumptions	From	To	Start Lat	Start Lon	Rhumbline Range (nm)	True Bearing (deg)	End Lat	End Lon	delta	q	deltapsi	earth radius in km at start latitude
Use 477 kts TGS and bearing to BITOD	IGARI	Deviation	6.937	103.585	15.90	58.81	7.074	103.813	0.00462	0.99253	0.00241	6377.83
Use radar bearing or use best fit depending on soonest allowable turn time	Last radar	18:25	6.550	96.330	27.13	288.25	6.691	95.899	0.00788	0.99333	0.00248	6377.86
Use same bearing as previous leg	18:25	Start Turn	6.691	95.899	21.90	288.25	6.806	95.550	0.00636	0.99307	0.00201	6377.85
Start Turn to End Turn (use delta lat/lon from Turn Calculator below)	Start Turn	End Turn	6.806	95.550			6.707	95.377				
Use true bearing out of Turn 1	End Turn	19:41	6.707	95.377	599.85	191.80	-3.062	93.333	0.17418	0.99828	-0.17080	6377.85
Use true bearing out of Turn 1	19:41	20:41	-3.062	93.333	508.74	191.93	-11.343	91.568	0.14772	0.99122	-0.14581	6378.08
Use true bearing out of Turn 1	20:41	21:41	-11.343	91.568	502.86	191.97	-19.528	89.766	0.14603	0.96299	-0.14835	6377.32
Use true bearing out of Turn 1	21:41	22:41	-19.528	89.766	489.82	192.02	-27.501	87.912	0.14228	0.91594	-0.15193	6375.77
Use true bearing out of Turn 1	22:41	0:11	-27.501	87.912	677.66	192.06	-38.534	85.093	0.19691	0.83611	-0.23031	6373.61
Use true bearing out of Turn 1	0:11	0:19	-38.534	85.093	59.61	191.98	-39.506	84.828	0.01733	0.77690	-0.02182	6369.88

Note: Magnetic Declination Formulas, Wind Tables, Turn Calculator, and Constants are Located Below on This Sheet !!

Table 9-2 Best-Fit Magnetic Track Route at 188 Degrees

MH370 Satellite Data Fitting Results
Dr. Bobby Ulich - 2014.07.28

Constant Magnetic Track Solution (Magnetic Track = 188.0 deg)

Time Stamp (UTC)	Notes	Aircraft Position: Geocentric Latitude (deg N)	Aircraft Position: Longitude (deg E)	Aircraft Position: Altitude (ft)	Leg From Previous Position (Assume Rhumbline): Range (nm)	Bearing (deg)	Average Ground Speed (kts)	Using Average of Winds at Leg Ends: Average True Air Speed (TAS) (kts)	Average True Heading (deg)	Wind at Aircraft Position: Direction (deg)	Speed (kts)	Magnetic Declination at Aircraft Position (West of North) (deg)	Magnetic Declination at Midpoint Between This Aircraft Position and Previous Position (West of North) (deg)	Average Magnetic Track During Segment By Averaging Start, Midpoint, and End Declinations (deg)	BTO from Inmarsat Log (microsec)	Handshake Arc Radius Calculated from BTO Using ATSB Equation, Ellipsoidal Earth, and Latitude-Dependent Average Earth Radius (nm)	Radial Error = Great Circle Distance of Aircraft from Sub-Satellite Point Minus Handshake Arc Radius (nm)	At Aircraft Position: rac = Ellipsoidal Earth Radius At This Aircraft Latitude (km)	Average Earth Radius Integrated From Approximate Sub-Satellite Point to This Aircraft Latitude (Used in Handshake Arc Radius Calculation) (km)	Satellite Position From ATSB Report: X (km)	Y (km)	Z (km)
2014/3/7 16:41:00	KLIA (VKL)	2.727	101.710	69						69	18	0.1			14840	2235.1	-0.6	6378.1	6378.2	18120.3	38081.2	748.7
2014/3/7 17:07:00	Last ACARS Data	5.29	102.80	35,000	167.3	23.0	386	398	24.65	67	15	0.2	0.2		15600	2307.4	0.0	6378.0	6378.3	18125.3	38079.7	958.0
2014/3/7 17:21:00	Arrive IGARI	6.94	103.59	35,000	109.5	25.4	469	478	26.29	68	9	0.4	0.3					6377.8				
2014/3/7 17:23:00	Estimated Diversion Position (en route BITOD)	7.07	103.81	35,000	15.9	58.8	477	486	58.98	68	9	0.4	0.4					6377.8				
2014/3/7 17:51:45	Circa Penang AP (VPG)	5.26	100.27	35,000	238.1	242.8	497	483	242.94	51	20	0.4	0.4					6378.0				
2014/3/7 18:02:00	Malaysian Military Radar - Pulau Perak	5.70	98.92	35,000	84.9	288.2	497	488	289.86	45	14	0.5	0.4					6377.9				
2014/3/7 18:22:12	Malaysian Military Radar - Final Position	6.55	96.33	35,000	163.1	288.3	485	481	289.15	50	3	0.7	0.6					6377.9				
2014/3/7 18:25:27	1st Handshake	6.68	95.95	35,000	24.2	288.3	446	444.9	288.59	50	3	0.8	0.7		12520	1906.5	1.3	6377.9	6378.3	18136.7	38071.8	1148.5
2014/3/7 18:35:51	Start of Final Turn	7.08	94.71	35,000	77.4	288.3	447	445.9	288.41	135	0	0.9	0.8					6377.8				
2014/3/7 18:37:32	End of Final Turn	6.98	94.56	35,000	12.3	237.8	440	439.5	237.72	135	0	0.9	0.9					6377.8				
2014/3/7 19:41:00	2nd Handshake	-0.86	93.60	35,000	474.7	187.0	449	445.9	185.365	83	25	1.1	0.8	187.91	11500	1757.5	-3.0	6378.1	6378.1	18145.1	38067.0	1206.3
2014/3/7 20:41:02	3rd Handshake	-8.30	92.79	35,000	449.9	186.2	450	446.1	183.708	88	14	2.4	1.6	187.88	11740	1796.2	-0.5	6377.7	6377.5	18152.1	38064.0	1159.7
2014/3/7 21:41:24	4th Handshake	-15.95	92.22	35,000	460.9	184.1	458	454.0	182.469	64	14	5.2	3.6	187.85	12780	1950.8	-2.2	6376.5	6376.5	18159.5	38061.3	1033.8
2014/3/7 22:41:19	5th Handshake	-23.50	92.13	35,000	453.5	180.7	454	450.6	182.049	272	34	9.7	7.1	188.00	14540	2190.1	0.2	6374.8	6375.1	18167.2	38058.3	837.2
2014/3/8 0:10:58	6th Handshake	-33.42	93.30	15,000	599.1	174.1	401	402	179.639	261	44	18.6	13.5	188.01	18040	2612.5	0.0	6371.7	6373.0	18177.5	38051.7	440.0
2014/3/8 0:19:29	7th Handshake	-34.27	93.51	5,000	52.1	168.9	367	370	175.205	254	38	19.6	19.1	188.01	18400	2652.8	0.0	6371.4	6372.8	18178.4	38050.8	390.5

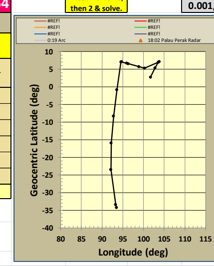
Average Ground Speed	452.2	AVG TAS = Last Radar to 22:41	448.9	KTS	AVG Magnetic Track Weighted by Range From End Turn to 00:19	187.94	RMS Radial Arc Error (nm) = All 7 Arcs: 1.50
RUN # MT#50 St Dev Ground Speed	4.0	STDEV TAS = Last Radar to 22:41	3.44	KTS	STDEV Magnetic Track About Desired G34 From End Turn to 00:19	0.096	MAX Radial Arc Error (nm) = All 7 Arcs: 2.99
		AVG TAS End Turn to 20:41	446.0	KTS			
		AVG TAS 20:41 to 22:41	452.3	KTS			

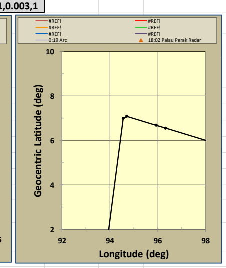
	Course	AVG TAS	StDev TAS	Turn Time	Lat	Lon	Max TAS	AVG GS	StDev GS
100,7,.0001	188.00	448.9	3.44	18:37:32	-34.27	93.51	454.0	452.2	4.0

AVG TAS	448.9	knots
STDEV TAS	3.44	knots
RMS Radial Error	1.50	nm
MAX Radial Error	2.99	nm
AVG Mag Track	187.94	deg
STDEV Mag Track	0.10	deg
MAX Mag Track Error	0.15	deg

	Mode=1 Gain	Objective Function To Be Minimized Using Solver	Weighting Coefficients	ST DEV	Objective Terms	MAX	Flag	StDev / Max Limit	Mode=2 Gain	Mode =1 Limit Scalar	Mode=1 Lowest Limit	Mode=2 Limit
100,7,0.1	50	Mag Track Error	0.001	0.096	0.000	0.152	1	0.10	0.0010	1.40	1.00	0.10
	7.0	Radial Error	0.0087	1.500	0.013	2.990	1	1.50	0.0030	1.40	1.10	1.50
	0.0000	True Air Speed	1.000	3.441	3.441	454	2	1500	1.00			
	Objective Cell to Minimize in Solver			3.454			Enter 1 & solve; then 2 & solve.		0.001,0.003,1			

Rad Err	TAS	Fitted Parameters									
1.31	445	24.18	Last Radar to 18:25 Arc	(nm)							
	446	77.39	18:25 to Start of Turn	(nm)							
	18:35:51	18.5974	Turn Start Time	(Hours)	>	18.4250	and	<	19.6167	hours	
Turn :	439	288.25	True Bearing Into 18:25 &Turn	(deg)		18:25:30	=Soonest Allowable Turn Start		19:37:00	=Latest Allowable Turn Start	
		188.00	Mag Track Out of Turn To 00:19	(deg)	Fitted Bearing To:		SAVED PARMS	Desired Mag Track	Actual Mag Track	Mag Track Error	
-2.99	446	474.72	186.97	End Turn to 19:41	(nm)	19:41	180.78	188.00	187.91	0.09	
-0.45	446	449.91	186.19	19:41 To 20:41	(nm)	20:41	181.74	188.00	187.88	0.12	
-2.20	454	460.99	184.13	20:41 To 21:41	(nm)	21:41	179.34	188.00	187.85	0.15	
0.24	451	453.58	180.67	21:41 To 22:41	(nm)	22:41	175.29	188.00	188.00	0.00	
0.00	402	599.21	174.07	22:41 To 0:11	(nm)	0:11	165.07	188.00	188.01	-0.01	
0.01	370	52.15	168.90	0:11 To 0:20	(nm)	0:19	157.39	188.00	188.01	-0.01	
		443.93	Turn Ground Speed	(kts)					MAX MT ERROR	0.1523	





Range and Bearing Calculations for Each Leg (Assume Rhumbline)

Assumptions	From	To	Start Lat	Start Lon	Rhumbline Range (nm)	True Bearing (deg)	End Lat	End Lon	delta	q	deltapsi	earth radius in km at start latitude
Use 477 kts TGS and bearing to BITOD	IGARI	Deviation	6.937	103.585	15.90	58.81	7.074	103.813	0.00462	0.99253	0.00241	6377.83
Use radar bearing or use best fit depending on soonest allowable turn time	Last radar	18:25	6.550	96.330	24.18	288.25	6.676	95.945	0.00702	0.99335	0.00221	6377.86
Use same bearing as previous leg	18:25	Start Turn	6.676	95.945	77.39	288.25	7.079	94.714	0.02247	0.99280	0.00709	6377.85
Start Turn to End Turn (use delta lat/lon from Turn Calculator below)	Start Turn	End Turn	7.079	94.714			6.983	94.560				
Use true bearing out of Turn 1	End Turn	19:41	6.983	94.560	474.72	186.97	-0.857	93.600	0.13785	0.99779	-0.13714	6377.82
Use true bearing out of Turn 1	19:41	20:41	-0.857	93.600	449.91	186.19	-8.298	92.790	0.13064	0.99610	-0.13039	6378.13
Use true bearing out of Turn 1	20:41	21:41	-8.298	92.790	460.99	184.13	-15.948	92.225	0.13387	0.97690	-0.13668	6377.70
Use true bearing out of Turn 1	21:41	22:41	-15.948	92.225	453.58	180.67	-23.496	92.131	0.13174	0.94049	-0.14007	6376.54
Use true bearing out of Turn 1	22:41	0:11	-23.496	92.131	599.21	174.07	-33.417	93.305	0.17408	0.87744	-0.19734	6374.76
Use true bearing out of Turn 1	0:11	0:19	-33.417	93.305	52.15	168.90	-34.269	93.506	0.01516	0.83056	-0.01791	6371.69

Note: Magnetic Declination Formulas, Wind Tables, Turn Calculator, and Constants are Located Below on This Sheet !!

Table 9-3 Best-Fit True Heading Route at 185 Degrees

MH370 Satellite Data Fitting Results
Dr. Bobby Ulich - 2014.07.28

Constant True Heading Solution (True Heading = 185.0 deg)

Time Stamp (UTC)	Notes	Aircraft Position: Geocentric Latitude (deg N)	Aircraft Position: Longitude (deg E)	Aircraft Position: Altitude (ft)	Leg From Previous Position (Assume Rhumbline): Range (nm)	Leg: Bearing (deg)	Leg: Average Ground Speed (kts)	Leg, Using Average of Winds at Leg Ends: Average True Air Speed (TAS) (kts)	Leg, Using Average of Winds at Leg Ends: Average True Heading (deg)	Wind at Aircraft Position: Direction (deg)	Wind at Aircraft Position: Speed (kts)	Magnetic Declination at Aircraft Position (West of North) (deg)	Magnetic Declination at Midpoint Between This Aircraft Position and Previous Position (West of North) (deg)	Average Magnetic Heading During Segment By Averaging Start, Midpoint, and End Declinations (deg)	BTO from Inmarsat Log (microsec)	Handshake Arc Radius Calculated from BTO Using ATSB Equation, Ellipsoidal Earth, and Latitude-Dependent Average Earth Radius (nm)	Radial Error = Great Circle Distance of Aircraft from Sub-Satellite Point Minus Handshake Arc Radius (nm)	At Aircraft Position: rac = Ellipsoidal Earth Radius At This Aircraft Latitude (km)	At Aircraft Position: Average Earth Radius Integrated From Approximate Sub-Satellite Point to This Aircraft Latitude (Used in Handshake Arc Radius Calculation) (km)	Satellite Position From ATSB Report: X (km)	Satellite Position: Y (km)	Satellite Position: Z (km)
2014/3/7 16:41:00	KLIA (VKL)	2.727	101.710	69						69	18	0.1			14840	2235.1	-0.6	6378.1	6378.2	18120.3	38081.2	748.7
2014/3/7 17:07:00	Last ACARS Data	5.29	102.80	35,000	167.3	23.0	386	398	24.65	67	15	0.2	0.2		15600	2307.4	0.0	6378.0	6378.3	18125.3	38079.7	958.0
2014/3/7 17:21:00	Arrive IGARI	6.94	103.59	35,000	109.5	25.4	469	478	26.29	68	9	0.4	0.3					6377.8				
2014/3/7 17:23:00	Estimated Diversion Position (en route BITOD)	7.07	103.81	35,000	15.9	58.8	477	486	58.98	68	9	0.4	0.4					6377.8				
2014/3/7 17:51:45	Circa Penang AP (VPG)	5.26	100.27	35,000	238.1	242.8	497	483	242.94	51	20	0.4	0.4					6378.0				
2014/3/7 18:02:00	Malaysian Military Radar - Pulau Perak	5.70	98.92	35,000	84.9	288.2	497	488	289.86	45	14	0.5	0.4					6377.9				
2014/3/7 18:22:12	Malaysian Military Radar - Final Position	6.55	96.33	35,000	163.1	288.3	485	481	289.15	50	3	0.7	0.6					6377.9				
2014/3/7 18:25:27	1st Handshake	6.68	95.92	35,000	25.6	288.3	473	471.8	288.57	50	3	0.8	0.7		12520	1906.5	0.00	6377.8	6378.3	18136.7	38071.8	1148.5
2014/3/7 18:35:06	Start of Final Turn	7.07	94.73	35,000	74.7	288.3	464	462.9	288.41	135	0	0.9	0.8					6377.8				
2014/3/7 18:36:48	End of Final Turn	6.96	94.56	35,000	14.0	237.3	494	493.8	237.23	135	0	0.9	0.9					6377.8				
2014/3/7 19:41:00	2nd Handshake	-1.27	93.61	35,000	498.2	186.6	466	462.9	185.062	83	25	1.1	0.9	186.01	11500	1757.5	-0.4	6378.1	6378.1	18145.1	38067.0	1206.3
2014/3/7 20:41:02	3rd Handshake	-8.90	92.64	35,000	461.9	187.2	462	458.3	184.860	92	13	2.6	1.7	186.66	11740	1796.2	2.5	6377.6	6377.4	18152.1	38064.0	1159.7
2014/3/7 21:41:24	4th Handshake	-16.73	91.75	35,000	473.6	186.3	471	468.5	184.963	73	10	5.8	3.9	189.05	12780	1950.6	-1.7	6376.4	6376.4	18159.5	38061.3	1033.8
2014/3/7 22:41:19	5th Handshake	-24.40	91.31	35,000	461.7	183.1	462	460.8	185.166	274	42	10.8	8.0	193.36	14540	2189.9	0.4	6374.5	6375.0	18167.2	38058.3	837.2
2014/3/8 0:10:58	6th Handshake	-35.01	91.21	15,000	637.0	180.5	426	434	185.049	241	31	22.5	15.8	201.44	18040	2612.1	0.0	6371.1	6372.6	18177.5	38051.7	440.0
2014/3/8 0:19:29	7th Handshake	-35.98	91.19	5,000	58.4	181.0	412	433	185.055	236	42	24.0	23.2	208.29	18400	2652.4	0.0	6370.8	6372.4	18178.4	38050.8	390.5

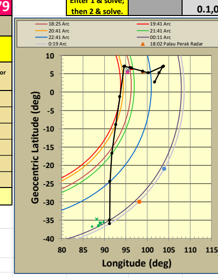
	Average Ground Speed	465.4	AVG TAS = Last Radar to 22:41	463.0	KTS	AVG True Heading Weighted by Range From End Turn to 00:19	185.02	RMS Radial Arc Error (nm) = All 7 Arcs: 1.14
RUN # TH#22	St Dev Ground Speed	4.3	STDEV TAS = Last Radar to 22:41	4.57	KTS	STDEV True Heading About Desired G34 From End Turn to 00:19	0.100	MAX Radial Arc Error (nm) = All 7 Arcs: 2.47
			AVG TAS End Turn to 20:41	460.7	KTS			
			AVG TAS 20:41 to 22:41	464.7	KTS			

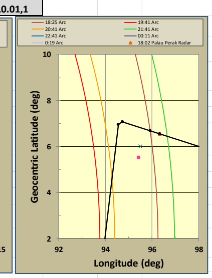
Course	AVG TAS	StDev TAS	Turn Time	Lat	Lon	Max TAS	AVG GS	StDev GS
185.00	463.0	4.57	18:36:48	-35.98	91.19	493.8	465.4	4.3

AVG TAS	463.0	knots
STDEV TAS	4.6	knots
RMS Radial Error	1.14	nm
MAX Radial Error	2.47	nm
AVG True Heading	185.02	deg
STDEV True Heading	0.10	deg
MAX True Heading Error	0.17	deg

	Mode=1 Gain	Objective Function To Be Minimized Using Solver	Weighting Coefficients	ST DEV	Objective Terms	MAX	Flag	StDev / Max Limit	Mode=2 Gain	Mode =1 Limit Scalar	Mode=1 Lowest Limit	Mode=2 Limit
100,7,0.01	100	True Heading Error	0.001	0.100	0.000	0.166	1	0.10	0.0010	1.40	1.00	0.10
	7.0	Radial Error	0.006	1.145	0.007	2.468	1	1.50	0.0030	1.40	1.10	1.50
	0.0000	True Air Speed	1.0000	4.571	4.571	494	2	1500	1.00			
	Objective Cell to Minimize in Solver				4.579		Enter 1 & solve; then 2 & solve.		0.1,0.01,1			

Rad Err (nm)	TAS (kts)	Fitted Parameters						
0.00	472	25.64	Range - Last Radar to 18:25 Arc	(nm)				
	463	74.65	Range - 18:25 to Start of Turn	(nm)				
	18:35:06	18.5851	Turn Start Time	(Hours)	> 18.4250 and < 19.6167 hours			
Turn :	494	288.25	True Bearing Into 18:25 &Turn (same as radar portion)	(deg)	18:25:30 =Soonest Allowable Turn Start		19:37:00 =Latest Allowable Turn Start	
		185.00	Input True Heading Out of Turn To 00:19	(deg)	Fitted Range & Bearing		Actual True Heading (deg)	True Heading Error (deg)
-0.36	463	498.21	186.61	End Turn to 19:41	(nm & deg)	19:41	185.06	-0.06
2.47	458	461.90	187.20	19:41 To 20:41	(nm & deg)	20:41	184.86	0.14
-1.68	468	473.61	186.28	20:41 To 21:41	(nm & deg)	21:41	184.96	0.04
0.37	461	461.81	183.09	21:41 To 22:41	(nm & deg)	22:41	185.17	-0.17
-0.01	434	637.17	180.47	22:41 To 0:11	(nm & deg)	0:11	185.05	-0.05
0.01	433	58.44	180.98	0:11 To 0:20	(nm & deg)	0:19	185.06	-0.06
		498.76	Turn Ground Speed	(kts)			MAX TH ERROR	0.166





Range and Bearing Calculations for Each Leg (Assume Rhumbline)

Assumptions	From	To	Start Lat	Start Lon	Rhumbline Range (nm)	True Bearing (deg)	End Lat	End Lon	delta	q	deltapsi	earth radius in km at start latitude
Use 477 kts TGS and bearing to BITOD	IGARI	Deviation	6.937	103.585	15.90	58.81	7.074	103.813	0.00462	0.99253	0.00241	6377.83
Use radar bearing or use best fit depending on soonest allowable turn time	Last radar	18:25	6.550	96.330	25.64	288.25	6.684	95.922	0.00745	0.99334	0.00235	6377.86
Use same bearing as previous leg	18:25	Start Turn	6.684	95.922	74.65	288.25	7.073	94.734	0.02168	0.99280	0.00684	6377.85
Start Turn to End Turn (use delta lat/lon from Turn Calculator below)	Start Turn	End Turn	7.073	94.734			6.962	94.561				
	End Turn	19:41	6.962	94.561	498.21	186.61	-1.272	93.606	0.14467	0.99790	-0.14401	6377.83
	19:41	20:41	-1.272	93.606	461.90	187.20	-8.895	92.638	0.13412	0.99532	-0.13369	6378.13
	20:41	21:41	-8.895	92.638	473.61	186.28	-16.728	91.753	0.13753	0.97427	-0.14032	6377.63
	21:41	22:41	-16.728	91.753	461.81	183.09	-24.402	91.309	0.13413	0.93538	-0.14319	6376.38
	22:41	0:11	-24.402	91.309	637.17	180.47	-35.008	91.208	0.18512	0.86654	-0.21362	6374.51
	0:11	0:19	-35.008	91.208	58.44	180.98	-35.981	91.188	0.01699	0.81415	-0.02086	6371.14

Note: Magnetic Declination Formulas, Wind Tables, Turn Calculator, and Constants are Located Below on This Sheet !!

Table 9-4 Best-Fit Magnetic Heading Route at 184 Degrees

MH370 Satellite Data Fitting Results
Dr. Bobby Ulich - 2014.07.28

Constant Magnetic Heading Solution (Mag Heading = 184.0 deg)

Time Stamp (UTC)	Notes	Aircraft Position: Geocentric Latitude (deg N)	Longitude (deg E)	Altitude (ft)	Leg From Previous Position (Assume Rhumbline): Range (nm)	Bearing (deg)	Average Ground Speed (kts)	Using Average of Winds at Leg Ends: Average True Air Speed (TAS) (kts)	Average True Heading (deg)	Wind at Aircraft Position: Direction (deg)	Speed (kts)	Magnetic Declination at Aircraft Position (West of North) (deg)	Magnetic Declination at Midpoint Between This Aircraft Position and Previous Position (West of North) (deg)	Average Magnetic Heading During Segment By Averaging Start, Midpoint, and End Declinations (deg)	BTO from Inmarsat Log (microsec)	Handshake Arc Radius Calculated from BTO Using ATSB Equation, Ellipsoidal Earth, and Latitude-Dependent Average Earth Radius (nm)	Radial Error = Great Circle Distance of Aircraft from Sub-Satellite Point Minus Handshake Arc Radius (nm)	At Aircraft Position: rac = Ellipsoidal Earth Radius At This Aircraft Latitude (km)	Average Earth Radius Integrated From Approximate Sub-Satellite Point to This Aircraft Latitude (Used in Handshake Arc Radius Calculation) (km)	Satellite Position From ATSB Report: X (km)	Y (km)	Z (km)
2014/3/7 16:41:00	KLIA (VKL)	2.727	101.710	69						69	18	0.1			14840	2235.1	-0.6	6378.1	6378.2	18120.3	38081.2	748.7
2014/3/7 17:07:00	Last ACARS Data	5.29	102.80	35,000	167.3	23.0	386	398	24.65	67	15	0.2	0.2		15600	2307.4	0.0	6378.0	6378.3	18125.3	38079.7	958.0
2014/3/7 17:21:00	Arrive IGARI	6.94	103.59	35,000	109.5	25.4	469	478	26.29	68	9	0.4	0.3					6377.8				
2014/3/7 17:23:00	Estimated Diversion Position (en route BITOD)	7.07	103.81	35,000	15.9	58.8	477	486	58.98	68	9	0.4	0.4					6377.8				
2014/3/7 17:51:45	Circa Penang AP (VPG)	5.26	100.27	35,000	238.1	242.8	497	483	242.94	51	20	0.4	0.4					6378.0				
2014/3/7 18:02:00	Malaysian Military Radar - Pulau Perak	5.70	98.92	35,000	84.9	288.2	497	488	289.86	45	14	0.5	0.4					6377.9				
2014/3/7 18:22:12	Malaysian Military Radar - Final Position	6.55	96.33	35,000	163.1	288.3	485	481	289.15	50	3	0.7	0.6					6377.9				
2014/3/7 18:25:27	1st Handshake	6.68	95.92	35,000	25.6	288.3	473	472	288.57	50	3	0.8	0.7		12520	1906.5	0.00	6377.8	6378.3	18136.7	38071.8	1148.5
2014/3/7 18:37:57	Start of Final Turn	7.16	94.46	35,000	91.9	288.3	441	440	288.27	180	3	0.9	0.8					6377.8				
2014/3/7 18:39:41	End of Final Turn	7.06	94.30	35,000	13.2	236.3	457	459	236.00	180	3	0.9	0.9					6377.8				
2014/3/7 19:41:00	2nd Handshake	-0.39	93.69	35,000	449.0	184.7	439	439	183.125	88	24	1.0	0.9	184.06	11500	1757.5	0.0	6378.1	6378.1	18145.1	38067.0	1206.3
2014/3/7 20:41:02	3rd Handshake	-7.62	93.06	35,000	436.1	184.9	436	434	182.399	88	14	2.2	1.5	183.96	11740	1796.3	2.9	6377.8	6377.6	18152.1	38064.0	1159.7
2014/3/7 21:41:24	4th Handshake	-15.09	92.72	35,000	449.3	182.6	447	443	180.833	68	15	4.6	3.2	184.16	12780	1950.9	-2.6	6376.7	6376.6	18159.5	38061.3	1033.8
2014/3/7 22:41:19	5th Handshake	-22.38	93.08	35,000	438.6	177.3	439	436	177.698	274	21	8.3	6.2	184.10	14540	2190.4	0.8	6375.1	6375.4	18167.2	38058.3	837.2
2014/3/8 0:10:58	6th Handshake	-31.39	95.59	15,000	557.6	166.1	373	364	172.845	271	68	14.4	11.0	184.10	18040	2613.0	0.0	6372.4	6373.5	18177.5	38051.7	440.0
2014/3/8 0:19:29	7th Handshake	-32.15	95.93	3,000	48.8	159.1	344	326	169.379	269	56	15.0	14.7	184.09	18400	2653.1	0.0	6372.1	6373.3	18178.4	38050.8	390.5

Average Ground Speed	440.8	St Dev Ground Speed	5.5
AVG TAS = Last Radar to 22:41	438.7 KTS	STDEV TAS = Last Radar to 22:41	5.46 KTS
AVG TAS End Turn to 20:41	436.6 KTS	AVG TAS 20:41 to 22:41	439.3 KTS
AVG Mag Heading Weighted by Range From End Turn to 00:19	184.08	STDEV Mag Heading About Command From End Turn to 00:19	0.100
RMS Radial Arc Error (nm) = All 7 Arcs	1.50	MAX Radial Arc Error (nm) = All 7 Arcs	2.89

Course	AVG TAS	StDev TAS	Turn Time	Lat	Lon	Max TAS	AVG GS	StDev GS
184.00	438.7	5.46	18:39:41	-32.15	95.93	471.8	440.8	5.5

RUN #	MHYYY	
AVG TAS	438.7	knots
STDEV TAS	5.5	knots
RMS Radial Error	1.50	nm
MAX Radial Error	2.9	nm
AVG Mag Heading	184.08	deg
STDEV Mag Heading	0.10	deg
MAX Mag Heading Error	0.16	deg

Mode=1 Gain	Objective Function To Be Minimized Using Solver	Weighting Coefficients	ST DEV	Objective Terms	MAX	Flag	StDev / Max Limit	Mode=2 Gain	Mode =1 Limit Scalar	Mode=1 Lowest Limit	Mode=2 Limit
100	Mag Heading Error	0.00	0.100	0.000	0.161	1	0.10	0.0010	1.40	1.00	0.10
7.0	Radial Error	0.0055	1.500	0.008	2.889	1	1.50	0.0030	1.40	1.10	1.50
0.0030	True Air Speed	1.000	5.461	5.461	472	2	1500	1.00			

100,7,0.01

Objective Cell to Minimize in Solver: 5.470

Enter 1 & solve; then 2 & solve. 0.1,0.01,1

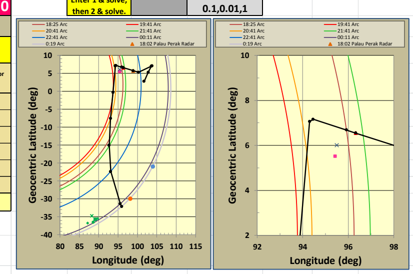
Fitted Parameters

Rad Err (nm)	TAS (kts)	Value	Parameter	Units
-0.001	472	25.64	Range - Last Radar to 18:25 Arc	(nm)
	440	91.87	Range - 18:25 to Start of Turn	(nm)
	18:37:57	18.6325	Turn Start Time	(Hours)
Turn :	459	288.25	True Bearing Into 18:25 &Turn (same as radar portion)	(deg)
		184.00	Input Mag Heading Out of Turn To 00:19	(deg)

Turn Start Time: > 18.4250 and < 19.6167 hours

18:25:30 =Soonest Allowable Turn Start; 19:37:00 =Latest Allowable Turn Start

Rad Err (nm)	TAS (kts)	Range	Bearing	Leg	Units	Fitted Range & Bearing To:	Actual Mag Heading (deg)	Mag Heading Error (deg)
-0.040	439	449.02	184.70	End Turn to 19:41	(nm & deg)	19:41	184.06	-0.06
2.889	434	436.08	184.90	19:41 To 20:41	(nm & deg)	20:41	183.96	0.04
-2.591	443	449.32	182.62	20:41 To 21:41	(nm & deg)	21:41	184.16	-0.16
0.831	436	438.69	177.29	21:41 To 22:41	(nm & deg)	22:41	184.10	-0.10
0.008	364	557.74	166.09	22:41 To 0:11	(nm & deg)	0:11	184.10	-0.10
-0.005	326	48.76	159.12	0:11 To 0:20	(nm & deg)	0:19	184.09	-0.09
		461.78		Turn Ground Speed	(kts)		MAX MH ERROR	0.161



Range and Bearing Calculations for Each Leg (Assume Rhumbline)

Assumptions	From	To	Start Lat	Start Lon	Rhumbline Range (nm)	True Bearing (deg)	End Lat	End Lon	delta	q	deltapsi	earth radius in km at start latitude
Use 477 kts TGS and bearing to BITOD	IGARI	Deviation	6.937	103.585	15.90	58.81	7.074	103.813	0.00462	0.99253	0.00241	6377.83
Use radar bearing or use best fit depending ton soonest allowable turn time	Last radar	18:25	6.550	96.330	25.64	288.25	6.684	95.922	0.00745	0.99334	0.00235	6377.86
Use same bearing as previous leg	18:25	Start Turn	6.684	95.922	91.87	288.25	7.162	94.460	0.02668	0.99271	0.00842	6377.85
Start Turn to End Turn (use delta lat/lon from Turn Calculator below)	Start Turn	End Turn	7.162	94.460			7.056	94.300				
	End Turn	19:41	7.056	94.300	449.02	184.70	-0.389	93.686	0.13039	0.99760	-0.13026	6377.82
	19:41	20:41	-0.389	93.686	436.08	184.90	-7.618	93.065	0.12662	0.99689	-0.12655	6378.14
	20:41	21:41	-7.618	93.065	449.32	182.62	-15.086	92.715	0.13047	0.97969	-0.13304	6377.76
	21:41	22:41	-15.086	92.715	438.69	177.29	-22.377	93.081	0.12741	0.94625	-0.13450	6376.70
	22:41	0:11	-22.377	93.081	557.74	166.09	-31.389	95.587	0.16203	0.89054	-0.17661	6375.06
	0:11	0:19	-31.389	95.587	48.76	159.12	-32.148	95.927	0.01417	0.85017	-0.01558	6372.37

Note: Magnetic Declination Formulas, Wind Tables, Turn Calculator, and Constants are Located Below on This Sheet !!

9.2 <u>Flight Management System Routes</u>

9.2.1 Great Circle at 192.1 Degrees

Table 9-5 is the EXCEL worksheet for a great circle route at 192.1 degrees initial bearing. This initial bearing was found by minimizing the variation of the true air speed while using great circle navigation through the Flight Management System. The principal purpose of this fit was to find possible waypoints that might have been entered using the FMS.

Table 9-5 Best-Fit Great Circle Route at 192.1 Degrees

MH370 Satellite Data Fitting Results
Dr. Bobby Ulich - 2014.08.24

Constant Great Circle Solution (Initial Bearing = 192.1 deg)

Time Stamp (UTC)	Notes	Aircraft Position: Geocentric Latitude (deg N)	Longitude (deg E)	Altitude (ft)	Leg From Previous Position (Assume Great Circle): Range (nm)	Initial Bearing (deg)	Average Ground Speed (kts)	Using Average of Winds at Leg Ends: Average True Air Speed (TAS) (kts)	Average True Heading (deg)	Wind at Aircraft Position: Direction (deg)	Speed (kts)	Magnetic Declination at Aircraft Position (West of North) (deg)	Magnetic Declination at Midpoint Between This Aircraft Position and Previous Position (West of North) (deg)	Average Magnetic Heading During Segment By Averaging Start, Midpoint, and End Declinations (deg)	BTO from Inmarsat Log (microsec)	Handshake Arc Radius Calculated from BTO Using ATSB Equation, Ellipsoidal Earth, and Latitude-Dependent Average Earth Radius (nm)	Radial Error = Great Circle Distance of Aircraft from Sub-Satellite Point Minus Handshake Arc Radius (nm)	At Aircraft Position: rac = Ellipsoidal Earth Radius At This Aircraft Latitude (km)	Average Earth Radius Integrated From Approximate Sub-Satellite Point to This Aircraft Latitude (Used in Handshake Arc Radius Calculation) (km)	Satellite Position From ATSB Report: X (km)	Y (km)	Z (km)
2014/3/7 16:41:00	KLIA (VKL)	2.727	101.710	69						69	18	0.1			14840	2235.1	-0.6	6378.1	6378.2	18120.3	38081.2	748.7
2014/3/7 17:07:00	Last ACARS Data	5.29	102.80	35,000	167.3	23.0	386	398	24.65	67	15	0.2	0.2		15600	2307.4	0.0	6378.0	6378.3	18125.3	38079.7	958.0
2014/3/7 17:21:00	Arrive IGARI	6.94	103.59	35,000	109.5	25.4	469	478	26.29	68	9	0.4	0.3					6377.8				
2014/3/7 17:23:00	Estimated Diversion Position (en route BITOD)	7.07	103.81	35,000	15.9	58.8	477	486	58.98	68	9	0.4	0.4					6377.8				
2014/3/7 17:51:45	Circa Penang AP (VPG)	5.26	100.27	35,000	238.1	242.8	497	483	242.94	51	20	0.4	0.4					6378.0				
2014/3/7 18:02:30	Malaysian Military Radar - Pulau Perak	5.70	98.92	35,000	84.9	288.2	474	465	289.95	45	14	0.5	0.4					6377.9				
2014/3/7 18:22:12	Malaysian Military Radar - Final Position	6.55	96.33	35,000	163.1	288.3	497	493	289.13	50	3	0.7	0.6					6377.9				
2014/3/7 18:25:27	1st Handshake	6.69	95.90	35,000	27.1	288.3	499	497.9	288.55	50	3	0.8	0.7		12520	1906.5	-1.3	6377.8	6378.3	18136.7	38071.8	1148.5
2014/3/7 18:27:24	Start of Final Turn	6.78	95.64	35,000	16.2	288.3	499	497.9	288.55	50	3	0.8	0.8					6377.8				
2014/3/7 18:29:00	End of Final Turn	6.68	95.47	35,000	13.3	240.3	499	497.9	240.27	135	0	0.8	0.8					6377.9				
2014/3/7 19:41:00	2nd Handshake	-3.06	93.37	35,000	604.1	192.12	503	497.9	190.798	74	24	1.3	0.8	191.79	11500	1757.4	-1.0	6378.1	6378.0	18145.1	38067.0	1206.3
2014/3/7 20:41:02	3rd Handshake	-11.34	91.56	35,000	507.4	192.17	507	497.9	190.502	58	11	3.6	2.2	192.91	11740	1795.8	-2.8	6377.3	6377.1	18152.1	38064.0	1159.7
2014/3/7 21:41:24	4th Handshake	-19.59	89.67	35,000	501.5	192.35	498	497.9	192.388	244	11	8.3	5.6	198.26	12780	1950.0	-2.0	6375.8	6375.9	18159.5	38061.3	1033.8
2014/3/7 22:41:19	5th Handshake	-27.69	87.66	35,000	488.1	192.70	489	497.9	196.062	272	51	16.2	11.8	208.15	14540	2189.1	-1.0	6373.6	6374.3	18167.2	38058.3	837.2
2014/3/8 0:10:58	6th Handshake	-38.81	84.49	15,000	653.6	193.22	437	457	198.196	242	39	34.0	24.0	222.90	18040	2611.1	0.000	6369.8	6371.7	18177.5	38051.7	440.0
2014/3/8 0:19:29	7th Handshake	-39.80	84.14	5,000	65.8	194.30	464	491	197.678	242	39	36.0	35.0	232.68	18400	2651.4	0.000	6369.4	6371.5	18178.4	38050.8	390.5

RUN #	*GCXXX*	
AVG TAS	*497.9*	*knots*
STDEV TAS	*0.00*	*knots*
RMS Radial Error	*1.50*	*nm*
MAX Radial Error	*2.82*	*nm*
Initial GC Bearing	*192.12*	*deg*
STDEV Bearing Error	*0.00*	*deg*
MAX Bearing Error	*0.00*	*deg*

Average Ground Speed	499.6	
St Dev Ground Speed	6.7	
AVG TAS = Last Radar to 22:41	497.9	KTS
STDEV TAS = Last Radar to 22:41	0.00	KTS
AVG TAS End Turn to 20:41	497.9	KTS
AVG TAS 20:41 to 22:41	497.9	KTS

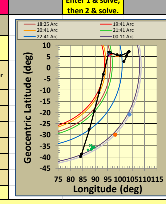
AVG True Track Weighted by Range From End Turn to 00:19	192.58	RMS Radial Arc Error (nm) = All 7 Arcs	1.50
STDEV True Track About Desired G34 From End Turn to 00:19	1.129	MAX Radial Arc Error (nm) = All 7 Arcs	2.82

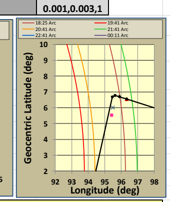
Course	AVG TAS	StDev TAS	Turn End Time	00:19 Lat	00:19 Lon	Max TAS	AVG GS	StDev GS
192.12	497.9	0.00	18:29:00	-39.80	84.14	497.9	499.6	6.7

	Mode=1 Gain	Objective Function To Be Minimized Using Solver	Weighting Coefficients	ST DEV	Objective Terms	MAX	Flag	StDev / Max Limit	Mode=2 Gain	Mode =1 Limit Scalar	Mode=1 Lowest Limit	Mode=2 Limit
100,7,0.1	100	Bearing Error	0.001	0.000	0.000	0.000	1	0.10	0.0010	1.40	1.00	0.10
	7.0	Radial Error	0.003	1.500	0.004	2.824	1	1.50	0.0030	1.40	1.10	1.50
	0.0030	True Air Speed	1.000	1.84E-08	0.000	498	2	1500	1.00			
	Objective Cell to Minimize in Solver				0.0045		Enter 1 & solve; then 2 & solve.		0.001,0.003,1			

Rad Err (nm)	TAS (kts)	Fitted Parameters								
-1.272	497.9	27.06	Last Radar to 18:25 Arc	(nm)						
	497.9	16.19	18:25 to Start of Turn	(nm)						
	18:27:24	18.4566	Turn Start Time	(Hours)	>	18.4250	and	<	19.6167	hours
Turn :	497.9	288.25	Bearing From Radar to 18:25 & to Turn	(deg)		18:25:30	=Soonest Allowable Turn Start		19:37:00	=Latest Allowable Turn Start
		192.12	Initial GC Bearing Out of Turn To 00:19	(deg)	Fitted Range & Bearing To:			Desired Initial Bearing	Actual Initial Bearing	Initial Bearing Error
-1.02	497.9	604.06	0.00	End Turn to 19:41	(nm & deg)	19:41		192.12	192.12	0.00
-2.82	497.9	507.45	0.00	19:41 To 20:41	(nm & deg)	20:41		192.17	192.17	0.00
-2.03	497.9	501.47	0.00	20:41 To 21:41	(nm & deg)	21:41		192.35	192.35	0.00
-1.00	497.9	488.08	0.00	21:41 To 22:41	(nm & deg)	22:41		192.70	192.70	0.00
0.00	457	653.64	0.00	22:41 To 0:11	(nm & deg)	0:11		193.22	193.22	0.00
0.00	491	65.82	0.00	0:11 To 0:20	(nm & deg)	0:19		194.30	194.30	0.00
		504.69	Turn Ground Speed		(kts)				MAX IB ERR	0.0000

Note: The fitted parameters after 18:22 are great circle ranges with a single given great circle initial bearing beginning at the end of the final turn. In this case the Great Circle range is used in the table above to figure ground speed and then air speed. In addition, in the table below, the GC final bearing for each leg after the final turn is used as the initial bearing for the next leg.





Range and Bearing Calculations for Each Leg : / **Rhumbline used for rough second point for better Earth radius average** / **Great Circle**

Assumptions	From	To	Start Geocentric Lat (deg)	Start Lon (deg)	Rhumbline Range (nm)	Bearing (deg)	End Lat	End Lon	delta	q	deltapsi	earth radius in km at start latitude	earth radius in km at approximate end latitude	Great Circle Range (nm)	Initial Bearing (deg)	"a" term for range computation	End Geocentric Latitude (deg)	End Longitude (deg)	Final Bearing (deg)
Use 477 kts TGS and bearing to BITOD	IGARI	Deviation	6.937	103.585	15.90	58.81	7.074	103.813	0.00462	0.99253	0.00241	6377.83							
Use radar bearing or use best fit depending on soonest allowable turn time	Last radar	18:25	6.550	96.330	27.06	288.25	6.691	95.900	0.00786	0.99333	0.00248	6377.86							
Use same bearing as previous leg	18:25	Start Turn	6.691	95.900	16.19	288.25	6.775	95.642	0.00470	0.99310	0.00148	6377.85							
Start Turn to End Turn (use delta lat/lon from Turn Calculator below)	Start Turn	End Turn	6.775	95.642			6.678	95.470											
Use true bearing out of Turn 1	End Turn	19:41	6.6776	95.4698	604.0622	192.1200	-3.148	93.356	0.17541	0.99830	-0.17179	6377.85	6378.07	604.0622	192.1200	0.007535733	-3.0586	93.3665	192.1667
Use true bearing out of Turn 1	19:41	20:41	-3.0586	93.3665	507.4500	192.1667	-11.311	91.571	0.14735	0.99126	-0.14531	6378.08	6377.32	507.4500	192.1667	0.005454979	-11.3380	91.5580	192.3520
Use true bearing out of Turn 1	20:41	21:41	-11.3380	91.5580	501.4712	192.3520	-19.489	89.705	0.14563	0.96310	-0.14771	6377.32	6375.77	501.4712	192.3520	0.005428193	-19.5905	89.6701	192.6979
Use true bearing out of Turn 1	21:41	22:41	-19.5905	89.6701	488.0830	192.6979	-27.515	87.720	0.14178	0.91568	-0.15104	6375.75	6373.60	488.0830	192.6979	0.00524171	-27.6881	87.6607	193.2225
Use true bearing out of Turn 1	22:41	0:11	-27.6881	87.6607	653.6445	193.2225	-38.282	84.685	0.18993	0.83661	-0.22101	6373.55	6369.97	653.6445	193.2225	0.009914881	-38.8081	84.4915	194.3028
Use true bearing out of Turn 1	0:11	0:19	-38.8081	84.4915	65.8177	194.3028	-39.871	84.141	0.01914	0.77338	-0.02398	6369.78	6369.39	65.8177	194.3028	8.11983E-05	-39.8040	84.1390	195.4319

Note: Magnetic Declination Formulas, Wind Tables, Turn Calculator, and Constants are Located Below on This Sheet !!

9.2.2 Great Circle Over Maimun Saleh Airport

Inspection of aeronautical charts identified the nearby waypoint for an approximately 192 degree initial bearing route. This waypoint is the Maimun Saleh Airport on Weh Island in the Andaman Sea. The data for this route are shown in Table 9-6.

Table 9-6 Great Circle Route Over Maimun Saleh Airport

MH370 Satellite Data Fitting Results
Dr. Bobby Ulich - 2014.09.03

Constant Speed Great Circle Solution (Initial Bearing = 192.4 deg Through Maimun Saleh)

Time Stamp (UTC)	Notes	Aircraft Position: Geocentric Latitude (deg N)	Geodetic Latitude (deg N)	Longitude (deg E)	Altitude (ft)	Leg From Previous Position (Assume Great Circle): Range (nm)	Initial Bearing (deg)	Average Ground Speed (kts)	Using Average of Winds at Leg Ends: Average True Air Speed (TAS) (kts)	Average True Heading (deg)	Wind at Aircraft Position: Direction (deg)	Speed (kts)	Magnetic Declination at Aircraft Position (West of North) (deg)	Magnetic Declination at Midpoint Between This Aircraft Position and Previous Position (West of North) (deg)	Average Magnetic Heading During Segment By Averaging Start, Midpoint, and End Declinations (deg)	BTO from Inmarsat Log (microsec)	Handshake Arc Radius Calculated from BTO Using ATSB Equation, Ellipsoidal Earth, and Average Latitude-Dependent Earth Radius (nm)	Radial Error = Great Circle Distance of Aircraft from Sub-Satellite Point Minus Handshake Arc Radius (nm)
2014/3/7 16:41:00	Takeoff from Kuala Lumpur Int. Airport (VKL)	2.709	2.727	101.710	69						69	18.2	0.1	0.1	0.1	14840	2235.1	-0.68
2014/3/7 16:44:00	Turn to IGARI	2.841	2.860	101.670	2600	8.3	343.17	165.9	168.3	349.3	69	18.2	0.1	0.1	349.5			
2014/3/7 17:06:43	Last ACARS Data	5.255	5.290	102.800	35,000	160.1	25.03	422.9	435.1	26.5	67	14.6	0.2	0.1	26.6	15600	2307.4	-0.16
2014/3/7 17:20:39	Arrive IGARI	6.891	6.937	103.585	35,000	108.9	25.51	469.1	478.0	26.4	68	8.8	0.4	0.3	26.7			
2014/3/7 17:21:37	Last Secondary Radar Data	6.974	7.020	103.680	35,000	7.5	48.73	469.7	478.0	49.1	68	8.8	0.4	0.4	49.4			
2014/3/7 17:22:20	Begin Diversion Turn-Around	7.023	7.070	103.760	35,000	5.6	57.97	469.3	478.0	58.2	68	8.8	0.4	0.4	58.5			
2014/3/7 17:24:39	End Diversion Turn-Around	7.212	7.260	103.680	35,000	19.3	337.19	499.9	499.9	338.2	68	8.8	0.4	0.4	338.6			
2014/3/7 17:36:38	Arrive Kota Bharu Airport (GOLUD)	6.243	6.285	102.278	35,000	102.0	235.18	510.6	500.1	235.1	54	12.5	0.3	0.4	235.5			
2014/3/7 17:52:37	Arrive Penang International Airport (VPG)	5.146	5.180	100.260	35,000	137.5	241.33	516.1	500.1	241.6	51	19.9	0.4	0.4	242.0			
2014/3/7 18:02:33	Military Radar - Pulau Perak	5.622	5.660	98.940	35,000	84.0	289.94	507.9	500.0	291.6	45	14.0	0.5	0.4	292.1			
2014/3/7 18:22:12	Military Radar - Final Position	6.507	6.550	96.330	35,000	164.8	288.81	503.2	499.4	289.7	50	3.0	0.7	0.6	290.3			
2014/3/7 18:25:27	1st Handshake	6.653	6.698	95.897	35,000	27.3	288.81	504.5	503.0	289.1	50	3.0	0.8	0.7	289.9	12520	1906.5	-1.76
2014/3/7 18:27:00	Start of Final Turn	6.723	6.768	95.689	35,000	13.1	288.81	504.5	503.0	289.1	50	3.0	0.8	0.8	289.9			
2014/3/7 18:28:36	End of Final Turn	6.626	6.670	95.514	35,000	13.5	240.82	505.9	503.0	240.9	50	3.0	0.8	0.8	241.7			
2014/3/7 18:34:23	Maimun Saleh Airport (SBG/WITN)	5.8351	5.87413	95.33967	35,000	48.7	192.36	506.0	503.0	191.7	81	10.2	0.8	0.8	192.5			
2014/3/7 19:41:00	2nd Handshake	-3.375	-3.398	93.331	35,000	566.6	192.34	510.2	503.0	190.6	74	23.9	1.4	1.4	191.8	11500	1757.4	-0.11
2014/3/7 20:41:02	3rd Handshake	-11.662	-11.738	91.492	35,000	509.9	192.30	509.7	503.0	190.4	90	12.2	3.8	2.3	192.9	11740	1795.8	1.66
2014/3/7 21:41:24	4th Handshake	-19.842	-19.965	89.563	35,000	504.0	192.54	501.0	503.0	192.3	244	10.8	8.5	5.8	198.4	12780	1950.0	2.45
2014/3/7 22:41:19	5th Handshake	-27.810	-27.969	87.476	35,000	492.2	193.06	492.9	503.0	196.2	269	47.8	16.4	12.0	208.5	14540	2189.1	-1.97
2014/3/8 0:10:58	6th Handshake	-39.069	-39.258	83.893	15,000	699.3	193.91	468.1	489.0	198.3	242	39.3	34.9	24.5	223.6	18040	2611.1	0.000
2014/3/8 0:19:29	7th Handshake	-40.046	-40.236	83.530	1,000	61.0	195.89	429.8	458.1	199.4	242	39.3	36.9	35.9	235.3	18400	2650.8	0.000

Constant Speed Great Circle Solution (Initial Bearing =

Time Stamp (UTC)	Notes	Aircraft Position: Geocentric Latitude (deg N)	Longitude (deg E)	Altitude (ft)
2014/3/7 16:41:00	Takeoff from Kuala Lumpur Int. Airport (VKL)	2.709	101.710	69
2014/3/7 16:44:00	Turn to IGARI	2.841	101.670	2,600
2014/3/7 17:06:43	Last ACARS Data	5.255	102.800	35,000
2014/3/7 17:20:39	Arrive IGARI	6.891	103.585	35,000
2014/3/7 17:21:37	Last Secondary Radar Data	6.974	103.680	35,000
2014/3/7 17:22:20	Begin Diversion Turn-Around	7.023	103.760	35,000
2014/3/7 17:24:39	End Diversion Turn-Around	7.212	103.680	35,000
2014/3/7 17:36:38	Arrive Kota Bharu Airport (GOLUD)	6.243	102.278	35,000
2014/3/7 17:52:37	Arrive Penang International Airport (VPG)	5.146	100.260	35,000
2014/3/7 18:02:33	Military Radar - Pulau Perak	5.622	98.940	35,000
2014/3/7 18:22:12	Military Radar - Final Position	6.507	96.330	35,000
2014/3/7 18:25:27	1st Handshake	6.653	95.897	35,000
2014/3/7 18:27:00	Start of Final Turn	6.723	95.689	35,000
2014/3/7 18:28:36	End of Final Turn	6.626	95.514	35,000
2014/3/7 18:34:23	Maimun Saleh Airport (SBG/WITN)	5.835	95.340	35,000
2014/3/7 19:41:00	2nd Handshake	-3.375	93.331	35,000
2014/3/7 20:41:02	3rd Handshake	-11.662	91.492	35,000
2014/3/7 21:41:24	4th Handshake	-19.842	89.563	35,000
2014/3/7 22:41:19	5th Handshake	-27.810	87.476	35,000
2014/3/8 0:10:58	6th Handshake	-39.069	83.893	15,000
2014/3/8 0:19:29	7th Handshake	-40.046	83.530	1,000

RUN #	GCXXX	
AVG TAS	502.96	knots
STDEV TAS	0.0000	knots
MAX TAS	502.96	knots
RMS Radial Error	1.50	nm
MAX Radial Error	2.45	nm
AVG Radial Error	0.04	nm
Initial GC Bearing	192.36	deg
STDEV Bearing Error	0.00	deg
MAX Bearing Error	0.00	deg

Average Ground Speed	503.7	AVG TAS = Last Radar to 22:41	503.0	KTS
St Dev Ground Speed	6.9	STDEV TAS = Last Radar to 22:41	0.00	KTS
MAX Ground Speed	510.2	AVG TAS End Turn to 20:41	503.0	KTS
MIN Ground Speed	492.9	AVG TAS 20:41 to 22:41	503.0	KTS

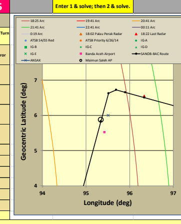
AVG Track Weighted by Range From End Turn to 00:19	192.95	RMS Radial Arc Error (nm) = All 7 Arcs	1.50
STDEV True Track About Desired G34 From End Turn to 00:19	0.000	MAX Radial Arc Error (nm) = All 7 Arcs	2.45

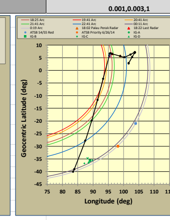
Course	AVG TAS	StDev TAS	End Turn Time	00:19 Lat	00:19 Lon	Max TAS	AVG GS	StDev GS	Initial Bearing
GC SBG/WITN	503.0	4.54E-07	18:28:36	-40.05	83.53	503.0	503.7	6.9	192.36

	Mode=1 Gain	Objective Function To Be Minimized Using Solver	Weighting Coefficients	ST DEV	Objective Terms	MAX	Flag	StDev / Max Limit	Mode=2 Gain	Mode =1 Limit Scalar	Mode=1 Lowest Limit	Mode=2 Limit
100,7,0.1	100	Bearing Error	0.001	0.000	0.000	0.000	1	0.10	0.0010	1.40	1.00	0.10
	7.0	Radial Error	0.003	1.500	0.005	2.455	1	1.50	0.0030	1.40	1.10	1.50
	0.0030	True Air Speed	1.000	4.54E-07	0.000	503	2	1500	1.00			
	Objective Cell to Minimize in Solver				0.0045		Enter 1 & solve; then 2 & solve.		0.001,0.003,1			

Rad Err (nm)	TAS (kts)	Fitted Parameters								
-1.757	503.0	27.329	Last Radar to 18:25 Arc	(nm)						
	503.0	13.08	18:25 to Start of Turn	(nm)						
	0:01:33	1.5556	Time Turn Starts After 18:25 Handshake	Minutes	>	18.4242	and	<	19.6167	hours
Turn :	503.0	288.81	Bearing From Radar to 18:25 & to Turn	(deg)		18:25:27	=Soonest Allowable Turn Start		19:37:00	=Latest Allowable Turn Start
		511.27	Ground Speed During Turn	(kts)			Desired Initial Bearing	Actual Initial Bearing	Initial Bearing Error	
		505.97	Ground Speed From End of Turn to SBG							
		GC Range	GC Init Bearing	From / To						
	503.0		193.00	Integer Bearing at end of turn	(deg)		192.36	192.36	0.00	
			192.36	Initial Bearing from End of Turn to SBG	(deg)					
-0.11	503.0	566.57	192.339	SBGto 19:41	(nm & deg)		192.34	192.34	0.00	
1.66	503.0	509.95	192.296	19:41 To 20:41	(nm & deg)		192.30	192.30	0.00	
2.45	503.0	504.01	192.537	20:41 To 21:41	(nm & deg)		192.54	192.54	0.00	
-1.97	503.0	492.19	193.062	21:41 To 22:41	(nm & deg)		193.06	193.06	0.00	
0.00	489	699.34	193.907	22:41 To 0:11	(nm & deg)		193.91	193.91	0.00	
0.00	458	61.00	195.891	0:11 To 0:19	(nm & deg)		195.89	195.89	0.00	
								MAX IB ERR	0.0000	

Note: The fitted parameters are great circle ranges with a single given great circle initial bearing beginning at the end of the turn. In this case the Great Circle range is used in the table above to figure ground speed and then air speed. In addition, in the table below, the GC final bearing for each leg after the final turn is used as the initial bearing for the next leg.





Range and Bearing Calculations for Each Leg : (Rhumbline / Great Circle)

Assumptions	From	To	Start Geocentric Lat (deg)	Start Lon (deg)	Rhumbline Range (nm)	Bearing (deg)	End Lat	End Lon	delta	q	deltapsi	earth radius in km at start latitude	earth radius in km at approximate end latitude	Great Circle Range (nm)	Initial Bearing (deg)	"a" term for range computation	End Geocentric Latitude (deg)	End Longitude (deg)	Final Bearing (deg)
Use 477 kts TGS and bearing to BITOD	IGARI	Deviation	6.891	103.585	7.79	58.81	6.958	103.697	0.00226	0.99271	0.00118	6377.83							
Use radar bearing	Last radar	18:25	6.507	96.330	27.33	288.81	6.653	95.897	0.00794	0.99341	0.00258	6377.86							
Use same bearing as previous leg	18:25	Start Turn	6.653	95.897	13.08	288.81	6.723	95.689	0.00380	0.99319	0.00123	6377.85							
Start Turn to End Turn (use delta lat/lon from Turn Calculator below)	Start Turn	End Turn	6.723	95.689	Great Circle Range	GC Initial Bearing	6.626	95.514											
Fly to Maimun Saleh on GC route from end of turn	End Turn	SBG	6.626	95.514		Geocentric:	5.83508	95.3397				6377.85	6377.92	48.66818	192.3577	4.99288E-05	5.8351	95.3397	192.3388
Continue great circle path past Maimun Saleh	SBG	19:41	5.8351	95.3397	566.5697	192.339	-3.373	93.323	0.16452	0.99869	-0.16093	6377.92	6378.06	566.5697	192.3388	0.006751202	-3.3750	93.3306	192.2955
Continue great circle path past Maimun Saleh	19:41	20:41	-3.3750	93.3306	509.9472	192.296	-11.664	91.507	0.14807	0.99050	-0.14606	6378.06	6377.27	509.9472	192.2955	0.005472113	-11.6618	91.4921	192.5367
Continue great circle path past Maimun Saleh	20:41	21:41	-11.6618	91.4921	504.0150	192.537	-19.848	89.599	0.14637	0.96148	-0.14860	6377.27	6375.69	504.0150	192.5367	0.00534775	-19.8416	89.5631	193.0617
Continue great circle path past Maimun Saleh	21:41	22:41	-19.8416	89.5631	492.1876	193.062	-27.821	87.537	0.14297	0.91371	-0.15242	6375.69	6373.51	492.1876	193.0617	0.005103138	-27.8097	87.4764	193.9068
Continue great circle path past Maimun Saleh	22:41	0:11	-27.8097	87.4764	699.3449	193.907	-39.112	84.112	0.20321	0.83172	-0.23717	6373.51	6369.67	699.3449	193.9068	0.010294683	-39.0691	83.8934	195.8913
Continue great circle path past Maimun Saleh	0:11	0:19	-39.0691	83.8934	61.0040	195.891	-40.047	83.532	0.01774	0.77096	-0.02213	6369.69	6369.33	61.0040	195.8913	7.86531E-05	-40.0460	83.5299	196.1228

Note: Magnetic Declination Formulas, Wind Tables, Turn Calculator, and Constants are Located Below on This Sheet !!

9.3 **BTO Calculations**

Constant Speed Great Circle Solution (Initial Bearing = 192.36 deg Through Maimun Saleh Airport)																								
Time Stamp	Notes	Aircraft Position			At Aircraft Position		Satellite Position From ATSB Report			Sub-Satellite Point (From ATSB X,Y,Z)				Handshake Arc Radii										
		Geocentric Latitude	Longitude	Altitude	rac = Ellipsoidal Earth Radius At This Aircraft Latitude	Average Earth Radius Integrated From Approximate Sub-Satellite Point to This Aircraft Latitude (Used in Handshake Arc Radius Calculation)	X	Y	Z	Geocentric Latitude	Longitude	rsat = Radius of Earth at Sub-Satellite Latitude	Hsat = Altitude of Satellite Above Earth's Surface	Satellite to Perth Slant Range	Rsat = Satellite to Aircraft Slant Range	Rsat + Hsat	rsat + Hsat	hac = Aircraft Altitude	rac + hac	Beta = Subtended Angle at Center of Earth	Handshake Arc Radii Calculated from BTO Using Integrated Average Ellipsoidal Earth Radius	BTO from Inmarsat Log	Handshake Arc Radius Calculated from BTO Using ATSB Equation, Ellipsoidal Earth, and Average Latitude-Dependent Earth Radius	Radial Error = Great Circle Distance of Aircraft from Sub-Satellite Point Minus Handshake Arc Radius
(UTC)		(deg N)	(deg E)	(ft)	(km)	(km)	(km)	(km)	(km)	(deg N)	(deg E)	(km)	(km)	(km)	km	(km)	(km)	(km)	(km)	(rad)	(km)	(microsec)	(nm)	(nm)
2014/3/7 16:41:00	Takeoff from Kuala Lumpur Int. Airport (VKL)	2.709	101.710	69	6378.1	6378.2	18120.3	38081.2	748.7	1.017	64.553	6378.1	35801.1	39227.3	37297.5	73098.6	42179.2	0.02	6378.1	0.64900	4139.4	14840	2235.1	-0.68
2014/3/7 16:44:00	Turn to IGARI	2.841	101.670	2,600	6378.1																			
2014/3/7 17:06:43	Last ACARS Data	5.255	102.800	35,000	6378.0	6378.3	18125.3	38079.7	958.0	1.301	64.546	6378.1	35806.1	39251.1	37387.7	73193.8	42184.2	10.67	6388.6	0.66999	4273.4	15600	2307.4	-0.16
2014/3/7 17:20:39	Arrive IGARI	6.891	103.585	35,000	6377.8																			
2014/3/7 17:21:37	Last Secondary Radar Data	6.974	103.680	35,000	6377.8																			
2014/3/7 17:22:20	Begin Diversion Turn-Around	7.023	103.760	35,000	6377.8																			
2014/3/7 17:24:39	End Diversion Turn-Around	7.212	103.680	35,000	6377.8																			
2014/3/7 17:36:38	Arrive Kota Bharu Airport (GOLUD)	6.243	102.278	35,000	6377.9																			
2014/3/7 17:52:37	Arrive Penang International Airport (VPG)	5.146	100.260	35,000	6378.0																			
2014/3/7 18:02:33	Military Radar - Pulau Perak	5.622	98.940	35,000	6377.9																			
2014/3/7 18:22:12	Military Radar - Final Position	6.507	96.330	35,000	6377.9																			
2014/3/7 18:25:27	1st Handshake	6.653	95.897	35,000	6377.9	6378.3	18136.7	38071.8	1148.5	1.560	64.528	6378.1	35808.6	39271.7	36905.5	72714.1	42186.7	10.67	6388.5	0.55357	3530.8	12520	1906.5	-1.76
2014/3/7 18:27:00	Start of Final Turn	6.723	95.689	35,000	6377.8																			
2014/3/7 18:28:36	End of Final Turn	6.626	95.514	35,000	6377.9																			
2014/3/7 18:34:23	Maimun Saleh Airport (SBG/WITN)	5.835	95.340	35,000	6377.9																			
2014/3/7 19:41:00	2nd Handshake	-3.375	93.331	35,000	6378.1	6377.9	18145.1	38067.0	1206.3	1.639	64.515	6378.1	35809.5	39278.6	36745.6	72555.1	42187.6	10.67	6388.7	0.51030	3254.7	11500	1757.4	-0.11
2014/3/7 20:41:02	3rd Handshake	-11.662	91.492	35,000	6377.3	6377.1	18152.1	38064.0	1159.7	1.575	64.504	6378.1	35808.5	39274.4	36785.8	72594.3	42186.6	10.67	6387.9	0.52152	3325.8	11740	1795.8	1.66
2014/3/7 21:41:24	4th Handshake	-19.842	89.563	35,000	6375.7	6375.8	18159.5	38061.3	1033.8	1.404	64.494	6378.1	35806.0	39261.7	36954.3	72760.3	42184.1	10.67	6386.4	0.56641	3611.4	12780	1950.0	2.45
2014/3/7 22:41:19	5th Handshake	-27.810	87.476	35,000	6373.5	6374.2	18167.2	38058.3	837.2	1.137	64.482	6378.1	35802.2	39241.8	37238.1	73040.3	42180.4	10.67	6384.2	0.63602	4054.2	14540	2189.1	-1.97
2014/3/8 0:10:58	6th Handshake	-39.069	83.893	15,000	6369.7	6371.6	18177.5	38051.7	440.0	0.598	64.466	6378.1	35794.7	39201.3	37803.2	73597.9	42172.8	4.57	6374.3	0.75894	4835.7	18040	2611.1	0.00
2014/3/8 0:19:29	7th Handshake	-40.046	83.530	1,000	6369.3	6371.4	18178.4	38050.8	390.5	0.531	64.464	6378.1	35793.8	39196.3	37862.2	73656.0	42171.9	0.30	6369.6	0.77052	4909.3	18400	2650.8	0.00

Table 9-7 Details of Handshake Arc Radius Calculations

Table 9-7 shows the details of the calculations I made to convert BTO data to handshake arc radii. I used the ATSB Report BTO equation, bias value, and satellite positions. The calculated arc radii match “ground truth” very closely for KLIA and the last ACARS data points (see the boxes shaded yellow). The boxes shaded orange show the radial errors for the 7 handshake arc times for the identified MH370 route over Maimun Saleh Airport.

9.4 BFO Calculations

Table 9-8 Details of BFO Calculations for the Maimun Saleh Airport Route

Time	Event	At Aircraft Position							BFO			Predicted BFO (Hz)							
		Geocentric Latitude	Geodetic Latitude	Longitude	Altitude	Ground Speed	Bearing	Rate of Climb	Measured	Predicted	Error	δf_{bias}	$\delta f_{sat} + \delta f_{AFC}$	ΔF_{down}	ΔF_{up}			$\Delta F_{rate\ of\ climb}$	δf_{comp}
UTC		degrees	degrees	degrees	feet	knots	degrees	feet / minute	Hz	Hz	Hz		From ATSB Table 4; includes eclipse effect		Total	Aircraft Toward Satellite	Satellite Toward Aircraft		Aircraft Terminal (AES) Compensation
2014/3/7 16:10:00	At Gate at KLIA Before Takeoff	2.709	2.727	101.710	69	0		0	86.8	89.3	2.4	149.5	30.5	-84.6	-6.1	0.0	-6.1	0.0	0.0
2014/3/7 16:41:00	Takeoff from Kuala Lumpur Int. Airport (VKL)	2.709	2.727	101.710	69	0		0	87.0	86.3	-0.7	149.5	23.5	-80.8	-5.9	0.0	-5.9	0.0	0.0
2014/3/7 16:44:00	Turn to IGARI	2.841	2.860	101.670	2,600	332	343.2	2,000		149.2		149.5	23.0	-79.4	160.7	166.5	-5.7	40.8	-145.4
2014/3/7 17:06:43	Last ACARS Data	5.255	5.290	102.800	35,000	469	25.0	0	131.5	129.1	-2.4	149.5	24.1	-71.9	-461.7	-458.5	-3.2	0.0	489.2
2014/3/7 17:20:39	Arrive IGARI	6.891	6.937	103.585	35,000	469	25.5	0		134.9		149.5	21.8	-66.0	-500.7	-498.8	-1.8	0.0	530.2
2014/3/7 17:21:37	Last Secondary Radar Data	6.974	7.020	103.680	35,000	469	48.7	0		126.9		149.5	21.6	-65.6	-779.1	-777.4	-1.8	0.0	800.6
2014/3/7 17:22:20	Begin Diversion Turn-Around	7.023	7.070	103.760	35,000	469	58.0	0		122.8		149.5	21.5	-65.2	-857.6	-855.9	-1.7	0.0	874.7
2014/3/7 17:24:39	End Diversion Turn-Around	7.212	7.260	103.680	35,000	500	337.2	0		138.7		149.5	21.0	-64.2	279.2	280.8	-1.6	0.0	-246.7
2014/3/7 17:36:38	Arrive Kota Bharu Airport (GOLUD)	6.243	6.285	102.278	35,000	513	235.2	0		84.1		149.5	19.0	-59.0	878.4	880.7	-2.3	0.0	-903.7
2014/3/7 17:52:37	Arrive Penang International Airport (VPG)	5.146	5.180	100.260	35,000	512	241.3	0		90.7		149.5	16.3	-52.0	875.0	877.8	-2.8	0.0	-898.2
2014/3/7 18:02:33	Military Radar - Pulau Perak	5.622	5.660	98.940	35,000	506	289.9	0		127.9		149.5	14.6	-47.5	829.7	832.0	-2.3	0.0	-818.3
2014/3/7 18:22:12	Military Radar - Final Position	6.507	6.550	96.330	35,000	504	288.8	0		134.2		149.5	11.1	-38.5	773.1	774.6	-1.5	0.0	-761.0
2014/3/7 18:25:27	1st Handshake	6.653	6.698	95.897	35,000	505	288.8	0	142.0	135.5	-6.5	149.5	10.7	-37.0	763.6	765.0	-1.4	0.0	-751.3
2014/3/7 18:27:00	Start of Final Turn	6.723	6.768	95.689	35,000	505	288.8	2,100		183.1		149.5	10.4	-36.3	759.6	760.9	-1.3	47.0	-747.1
2014/3/7 18:28:36	End of Final Turn	6.626	6.670	95.514	35,000	506	240.8	2,100		150.1		149.5	10.2	-33.4	790.8	792.1	-1.3	47.1	-814.0
2014/3/7 18:34:23	Maimun Saleh Airport (SBG/WITN)	5.835	5.874	95.340	35,000	508	192.4	0	87.8	91.6	3.8	149.5	9.3	-22.3	275.3	276.4	-1.1	0.0	-320.2
2014/3/7 19:41:00	2nd Handshake	-3.375	-3.398	93.331	35,000	510	192.3	0	111.0	101.7	-9.3	149.5	-0.5	-0.5	40.4	40.9	-0.5	0.0	-87.2
2014/3/7 20:41:02	3rd Handshake	-11.662	-11.738	91.492	35,000	505	192.3	0	141.0	140.8	-0.2	149.5	-1.5	28.7	-165.8	-172.9	7.0	0.0	130.0
2014/3/7 21:41:24	4th Handshake	-19.842	-19.965	89.563	35,000	501	192.5	0	168.0	170.6	2.6	149.5	-18.0	55.3	-353.0	-372.4	19.4	0.0	336.8
2014/3/7 22:41:19	5th Handshake	-27.810	-27.969	87.476	35,000	493	193.1	0	204.0	207.6	3.6	149.5	-28.5	77.8	-512.9	-547.5	34.6	0.0	521.7
2014/3/8 00:10:58	6th Handshake	-39.069	-39.258	83.893	15,000	449	193.9	0	252.0	258.6	6.6	149.5	-37.7	100.6	-658.2	-714.3	56.2	0.0	704.5
2014/3/8 00:19:29	7th Handshake	-40.046	-40.236	83.530	1,000	400	195.9	-4,676	182.0	182.0		149.5	-37.8	102.2	-574.3	-632.3	58.0	-82.3	624.8
2014/3/8 00:19:37	Extrapolated with 8 Seconds More Flight Time	-40.061	-40.251	83.524	100	400	195.9	-15,132	-2.0	-2.0		149.5	-37.8	102.2	-574.6	-632.5	58.0	-266.3	625.0